# **Control of Mobile Robots**

# PROF. BASCETTA

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## NAME:

UNIVERSITY ID NUMBER:

#### SIGNATURE:

#### Warnings

- This file consists of 8 pages (including cover).
- During the exam you are not allowed to exit the room for any other reason than handing your work or withdrawing from the exam.
- You are not allowed to withdraw from the exam during the first 30 minutes.
- During the exam you are not allowed to consult books or any kind of notes.
- You are not allowed to use calculators with graphic display.
- Solutions and answers can be given either in English or in Italian.
- Solutions and answers must be given **exclusively in the reserved space**. Only in the case of corrections, or if the space is not sufficient, use the back of the front cover.
- The clarity and the order of the answers will be considered in the evaluation.
- At the end of the test you have to **hand this file only**. Every other sheet you may hand will not be taken into consideration.

#### EXERCISE 1

Consider a car-trailer system, constituted by a trailer equipped with a fixed wheel and a car equipped with two steering wheels, shown in the figure below.



1. Determine the configuration vector and show the configuration variables on the figure above.

2. Derive the kinematic constraints that allow to determine the kinematic model of the car-trailer system, and write them in Pfaffian form.

3. Consider now the car without the trailer, and assume the rear wheel is fixed while the front is steerable. The car velocity and front steering position are constrained as follows

$$0 \le v \le \bar{v} \qquad \bar{\phi}_m \le \phi \le \bar{\phi}_M$$

How should we limit the linear and angular velocity of the canonical model, in order to be consistent with these constraints?

#### EXERCISE 2

As a result of an experimental campaign, a tire lateral force has been characterised by the following two force-slip curves (on the right you can see a close-up at slow slip angles)



1. Determine the values of the cornering stiffness and of the static friction, and write the analytical expression of a linear saturated model that fits the curves at the two different loads.

2. The vehicle is driving along a curve with  $F_z = 935$  N and  $\alpha = 20$  deg. What is the value of  $F_y$  obtained using the linear saturated model in this conditions? How does this value change if the longitudinal force  $F_x$  changes from 0 N to 100 N?

3. Consider the following tire models: linear, linear saturated, Fiala, Pacejka. Which is the appropriate model to design a control system (consider different operating conditions)? Which is the appropriate model to simulate a vehicle to test a control system? Clearly motivate the answer.

#### EXERCISE 3

1. Two important parts of an autonomous navigation system are the local and the global planner. What are the most important characteristics of these two functionalities?

2. Explain what are the advantages of structuring the navigation system in a hierarchical way, separating local from global planning. 3. Consider now as global planner RRT<sup>\*</sup>. Write the pseudocode of the rewire procedure, and explain the role of this procedure inside the RRT<sup>\*</sup> algorithm and its connection with optimality.

## **ESERCIZIO** 4

1. Consider the kinematic model of a mobile robot  $\dot{\mathbf{q}} = G(\mathbf{q}) \mathbf{u}$ . With reference to this model, give the definition of small-time local accessibility and small-time local controllability.

2. With reference to the kinematic model  $\dot{\mathbf{q}} = G(\mathbf{q})\mathbf{u}$ , give a condition to assess small-time local accessibility and small-time local controllability.

3. Using an example, explain how small-time local accessibility and small-time local controllability properties affect the mobility of a robot in an obstacle-free and in a non obstacle-free environment.