EXERCISE 1

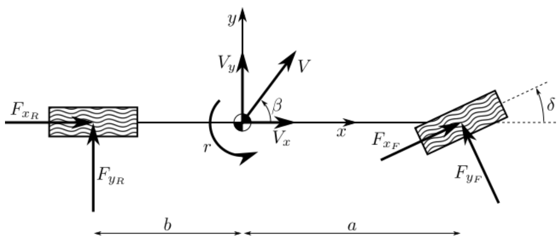
1. Write the expression of the two constraints that characterize a bicycle robot in Pfaffian form, and compute a basis of Null $\left(A^T\left(\mathbf{q}\right)\right)$.

2. Write the kinematic model of a rear-wheel drive bicycle, assuming the configuration is described by $[x\, y\, \theta]$ and the inputs are the linear velocity and the steering angle.

3. Show that (using a mathematical proof) the two constraints considered in step 1 are nonholonomic constraints.

EXERCISE 2

1. With reference to the following picture



write the Newton equations of the translational motion of the center of mass of the bicycle, and the Euler equation of the rotational motion, both referred to the body frame.

2. Consider now a bicycle with front and rear steering wheel. Write again the Newton equations of the translational motion of the center of mass of the bicycle, and the Euler equation of the rotational motion, both referred to the body frame.

3. During a curve the front wheel of a bicycle is subject to a longitudinal force $\bar{F}_{x_F} = 0.4\,\bar{F}_{z_F}$, and a lateral force $\bar{F}_{y_F} = 0.3\,\bar{F}_{z_F}$, where $\bar{F}_{z_F}$ is the vertical load on the wheel. Assume a unitary friction coefficient ($\mu = 1$).
Keeping constant the longitudinal force and the vertical load, which is the maximum value achievable by the lateral force?
Keeping constant the longitudinal force and doubling the vertical load, how does the maximum value change?

ESERCIZIO 3

1. Write the formal definition of the following problems:
   (a) feasible path planning;
   (b) optimal path planning;
   (c) optimal kinodynamic trajectory planning.

2. Consider kinodynamic RRT* planner applied to the kinematic model of a unicycle robot with the aim of minimizing the duration of the trajectory while penalizing the control effort.
   Write the problem that must be solved to compute an edge of the tree.

3. With reference to RRT*, explain the rewire procedure.
   Using an example, show how the rewire procedure modifies the tree.

ESERCIZIO 4

1. Consider the kinematic model of a unicycle robot, and a point P at a distance p from the wheel contact point along the direction of the velocity vector. Write the expression of the feedback linearizing controller and draw the block diagram of the system composed by the robot model and the controller.

2. Show that, applying the feedback linearizing controller the kinematic model of the unicycle is reduced to two independent integrators, i.e.,

$$\dot{x}_P = v_{xP} \qquad \dot{y}_P = v_{yP}$$

3. An experiment is executed on the real robot, performing a step response first on v_{xP} (with $v_{yP} = 0$), and then on v_{yP} (with $v_{xP} = 0$). Due to unmodelled dynamics, the two step responses appears as the response of a first order system (instead of an integrator), with unitary gain and a settling time of 0.05 seconds.
   Design and tune a trajectory tracking controller. Motivate how you select the crossover frequency.