

# Automatic Control A

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## Warnings:

- This file consists of **8** pages (including cover). All the pages should be signed.
- During the exam you are not allowed to exit the room for any other reason than handing your work or withdrawing from the exam.
- You are not allowed to withdraw from the exam during the first 30 minutes.
- During the exam you are not allowed to consult books or any kind of notes.
- You are not allowed to use calculators with graphic display.
- Solutions and answers can be given **either in English or in Italian**.
- Solutions and answers must be given **exclusively in the reserved space**. Only in the case of corrections, or if the space is not sufficient, use the back of the front cover.
- The clarity and the order of the answers will be considered in the evaluation.
- At the end of the test you have to **hand this file only**. Every other sheet you may hand will not be taken into consideration.

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**Use this page ONLY in case of corrections or if the space reserved for some answers turned out to be insufficient**

**Exercise 1**

Consider the following dynamical system

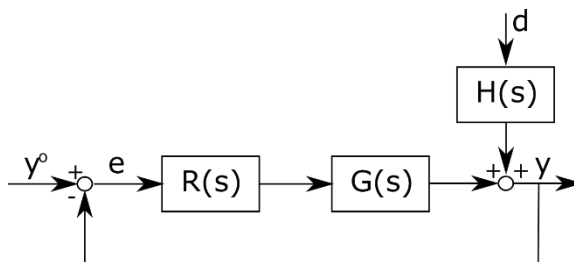
$$\begin{cases} \dot{x}_1 = -x_1 + 5x_2 + u \\ \dot{x}_2 = \alpha x_1 + u \\ \dot{x}_3 = x_1 + x_2 - 3x_3 \\ y = x_3 \end{cases}$$

**1.1** Find the values of parameter  $\alpha \in \mathbb{R}$  for which the system is asymptotically stable.

**1.2** Find the values of parameter  $\alpha \in \mathbb{R}$  for which the system is completely controllable and completely observable.

**Exercise 2**

Consider the following control system



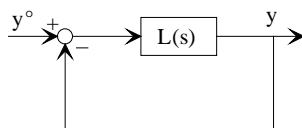
where  $G(s) = 10 \frac{10-s}{s(10+s)}$  and  $H(s) = \frac{10}{1+s}$ .

Compute the transfer function  $R(s)$  of a PD with filter on the derivative action in such a way that:

- $|e_\infty| = 0$  for  $y^o(t) = \text{sca}(t)$  and  $d(t) = \text{sca}(t)$ ;
- the phase margin  $\varphi_m$  is greater or equal to  $70^\circ$ ;
- the crossover frequency  $\omega_c$  is greater or equal to  $1 \text{ rad/s}$ .

**Exercise 3**

Consider the following closed-loop system



where  $L(s) = \rho \frac{s}{(s+1)^2(s-1)}$ .

**3.1** Sketch the direct and inverse root loci.

**3.2** Using the previous root loci, find the values of  $\rho$  for which the closed-loop system is asymptotically stable.

**Exercise 4**

Consider the following discrete time dynamical system

$$G(z) = \frac{z-1}{(z+2)(z+0.5)}.$$

**4.1** Compute the gain and type of the transfer function.

**4.2** Is the discrete time system stable, unstable or asymptotically stable?

**4.3** Compute the initial value and, if possible, the final value of the unit step response.

**4.4** Compute the first four samples of the unit step response.

**Exercise 5**

**5.1** Explain what is meant with kinematic and dynamic scaling of a trajectory, and write the general expression of a trajectory in normalized form that is used in such scaling.

**5.2** The parametric form of a harmonic trajectory is given by

$$\sigma(\tau) = \frac{1}{2}(1 - \cos(\pi\tau))$$

Find the expressions of the maximum velocity and maximum acceleration for such trajectory in terms of the positioning time  $T$  and the total displacement  $h$ .

**5.3** Consider the design of a harmonic trajectory from  $q_i = 10^\circ$  to  $q_f = 50^\circ$ , with  $\dot{q}_{max} = 30^\circ/s$  and  $\ddot{q}_{max} = 80^\circ/s^2$ . Find the minimum positioning time.

**Exercise 6**

**6.1** What is a successive-approximation ADC? Sketch the functional diagram that shows how this converter works, and list and describe the steps of the conversion process.

**6.2** Assuming that the full scale range of the ADC is  $10\text{ V}$  and the voltage resolution required by the application is  $10\text{ mV}$ , which is the minimum number of bits of the ADC in order to satisfy the requirement?

**6.3** Assuming that the harmonics of the analog signal to be converted span the range  $0\text{-}10\text{ kHz}$  and the conversion interval is  $1\ \mu\text{s}$ , is the sample-and-hold circuit required? Why?