Control of Mobile Robots

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Warnings

- This file consists of **8** pages (including cover).
- During the exam you are not allowed to exit the room for any other reason than handing your work or withdrawing from the exam.
- You are not allowed to withdraw from the exam during the first 30 minutes.
- During the exam you are not allowed to consult books or any kind of notes.
- You are not allowed to use calculators with graphic display.
- Solutions and answers can be given **either in English or in Italian**.
- Solutions and answers must be given **exclusively in the reserved space**. Only in the case of corrections, or if the space is not sufficient, use the back of the front cover.
- The clarity and the order of the answers will be considered in the evaluation.
- At the end of the test you have to **hand this file only**. Every other sheet you may hand will not be taken into consideration.

EXERCISE 1

1. Given a kinematic constraint in Pfaffian form

$$X(\mathbf{q})\,\dot{x} + Y(\mathbf{q})\,\dot{y} + Z(\mathbf{q})\,\dot{z} = 0$$

   where $\mathbf{q} = \begin{bmatrix} x & y & z \end{bmatrix}$ is the configuration vector. Illustrate the necessary and sufficient condition for this constraint to be holonomic.

2. Consider the following kinematic constraints

$$\dot{q}_1 + q_1\dot{q}_2 - \dot{q}_3 = 0 \qquad q_1\dot{q}_1 - q_2^2\dot{q}_3 = 0$$

   where $\mathbf{q} \in \mathbb{R}^3$ is the configuration vector. Determine, using the necessary and sufficient condition, if each of these constraints by itself is holonomic or nonholonomic.

3. Consider two mobile robots, whose configurations are described by $\mathbf{q} \in \mathbb{R}^3$, and whose motion is described by $\dot{q}_1 + q_1\dot{q}_2 - \dot{q}_3 = 0$ for the first robot, and $q_1\dot{q}_1 - q_2^2\dot{q}_3 = 0$ for the second robot.
   Does the following kinematic model

$$\dot{\mathbf{q}} = \begin{bmatrix} -q_1 \\ 1 \\ 0 \end{bmatrix} u_1 + \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} u_2$$

   describe the motion of the first robot, and the following kinematic model

$$\dot{\mathbf{q}} = \begin{bmatrix} q_2^2 \\ 0 \\ q_1 \end{bmatrix} u_1 + \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} u_2$$

   the motion of the second robot?
   Clearly motivate the answer.

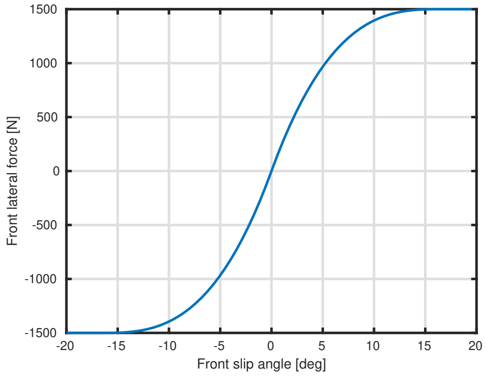
EXERCISE 2

1. Consider a bicycle described by the following simplified single-track model

$$\dot{\beta} = -\frac{C_{\alpha F} + C_{\alpha R}}{mV}\beta + \left(\frac{bC_{\alpha R} - aC_{\alpha F}}{mV^2} - 1\right) r + \frac{C_{\alpha F}}{mV}\delta$$
$$\dot{r} = \frac{bC_{\alpha R} - aC_{\alpha F}}{I_z}\beta - \frac{a^2C_{\alpha F} + b^2C_{\alpha R}}{I_zV} r + \frac{aC_{\alpha F}}{I_z}\delta$$

   List and explain all the assumptions under which this model is derived.
   Explain what are the conditions that allows to neglect wheel and actuator dynamics.

2. As a result of an experimental campaign, a tire lateral force has been characterised by the following picture



Assuming that the wheel normal force is equal to 2500 N:

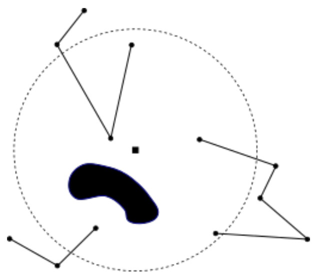
- determine the cornering stiffness and the static friction coefficient;
- write the analytical expression describing the force in the picture using the Fiala model;
- write the analytical expression describing the force in the picture using a linear model including saturation.

3. Consider a vehicle driving in a racetrack (for our purposes the vehicle is described by a single-track model). During i-th lap the lateral force of the front tyre, when the vehicle is performing the 4-th curve, is equal to the maximum available force (i.e., μF_{z_f}). How can the vehicle, during $(i+1)$-th lap, increase the velocity at which the same curve is performed without decreasing the longitudinal force?

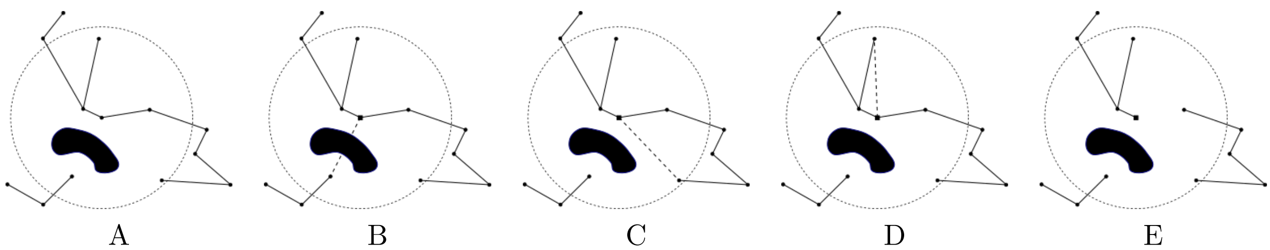
EXERCISE 3

1. Write and explain the pseudocode of the algorithm to construct the probabilistic roadmap used by PRM.

2. Consider the i-th iteration of PRM as depicted in the figure below



where the black square is the current $\mathbf{q}_{rand}$, the black blob region is an obstacle, the black dashed circle centred in $\mathbf{q}_{rand}$ is the region defining the Near set, and black dots and segments are nodes and edges in the actual V and E sets, respectively.
Put the following pictures in the correct order, according to the execution of the algorithm across the nodes belonging to the Near set (black dashed edges represent connection attempts that are discarded).



3. How could the path resulting from PRM be used to speed up the computation of RRT*? Clearly motivate the answer.

EXERCISE 4

Consider the design of the trajectory tracking controller for a robot modelled using the following rear-wheel drive bicycle kinematic model

$$\begin{aligned}\dot{x} &= v\cos\theta\\ \dot{y} &= v\sin\theta\\ \dot{\theta} &= v\frac{\tan\phi}{\ell}\\ \dot{\phi} &= \omega\end{aligned}$$

The controller design is based on the canonical simplified model.

1. Explain if and how the rear-wheel drive bicycle kinematic model can be reduced to the canonical simplified model, showing the relations to transform the inputs of the canonical simplified model to the ones of the rear-wheel drive bicycle kinematic model.

2. The actuators of the robot are characterised by the limitations $|\phi(t)| \leq \bar{\phi}$ and $0 \leq v(t) \leq \bar{v}$. How should we limit the linear and angular velocity of the canonical model, in order to be consistent with the robot limitations? Show how these constraints are derived.

3. Draw the Simulink diagram required to generate the code for a ROS node implementing the trajectory tracking controller, assuming that a block computing the control action (whose inputs are the actual and desired robot position, and whose outputs are the linear and angular velocity of the canonical simplified model) is given. In particular, specify:

   - the ROS messages used to interface the controller with the planner and the robot;
   - the aim and the implementation of each block in the diagram (excluding the one computing the control action previously mentioned).