Control of Mobile Robots

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Warnings

- This file consists of **8** pages (including cover).

- During the exam you are not allowed to exit the room for any other reason than handing your work or withdrawing from the exam.

- You are not allowed to withdraw from the exam during the first 30 minutes.

- During the exam you are not allowed to consult books or any kind of notes.

- You are not allowed to use calculators with graphic display.

- Solutions and answers can be given **either in English or in Italian**.

- Solutions and answers must be given **exclusively in the reserved space**. Only in the case of corrections, or if the space is not sufficient, use the back of the front cover.

- The clarity and the order of the answers will be considered in the evaluation.

- At the end of the test you have to **hand this file only**. Every other sheet you may hand will not be taken into consideration.

EXERCISE 1

1. Given a kinematic constraint in Pfaffian form

$$X(\mathbf{q})\dot{x} + Y(\mathbf{q})\dot{y} + Z(\mathbf{q})\dot{z} = 0$$

where $\mathbf{q} = \begin{bmatrix} x & y & z \end{bmatrix}$ is the configuration vector. Illustrate the necessary and sufficient condition for this constraint to be holonomic.

2. Consider the following kinematic constraints

$$\dot{q}_1 + q_1\dot{q}_2 + \dot{q}_3 = 0 \qquad \dot{q}_1 + \dot{q}_2 + q_1\dot{q}_3 = 0$$

where $\mathbf{q} \in \mathbb{R}^3$ is the configuration vector. Verify, using a formal demonstration, if each of these constraints by itself is holonomic or nonholonomic.

3. Consider the system of two constraints

$$\dot{q}_1 + q_1\dot{q}_2 + \dot{q}_3 = 0 \qquad \dot{q}_1 + \dot{q}_2 + q_1\dot{q}_3 = 0$$

Does the holonomicity/nonholonomicity of each constraint by itself imply the holonomicity/non-holonomicity of the system of constraints? Is the system of two constraints integrable?

EXERCISE 2

1. Consider a unicycle robot carrying a box full of sand, centred with respect to the robot center of gravity. The robot mass and yaw moment of inertia are equal to 30 Kg and 0.8 Kgm^2, when the box is empty, and 60 Kg and 1.6 Kgm^2, when the box is full. During the motion the box is leaking sand, and the mass and yaw moment of inertia change with time according to the following relations

$$M(t) = 60 - 0.5t \qquad I(t) = \frac{8}{5} - \frac{t}{75}$$

Write the dynamic model of the robot, assuming that there is no relative motion between the sand and the box and that the change of mass is slow compared to the motion of the robot.

2. The robot tire lateral force-slip relations are modelled using a piecewise constant model

$$F_y = \begin{cases} C_\alpha \alpha & \alpha \leq \alpha_{lin} \\ F_{y_{max}} & \alpha > \alpha_{lin} \end{cases}$$

where C_α is the cornering stiffness, α the slip angle, and $\alpha_{lin} = 0.1$ rad. The friction coefficient is $\mu = 0.5$.

Write the lateral force-slip relation at $t_1 = 0$ and $t_2 = 30$.

3. Consider that the robot is equipped with wheel velocity controllers tuned for the case of empty box. For the design of a model-based trajectory tracking controller, assuming either the linear and angular velocity or the wheel torque are available to the controller, is it better to consider the kinematic or dynamic model of the robot? Clearly motivate the answer.

EXERCISE 3

1. Consider a unicycle mobile robot. Selecting as flat outputs $z_1 = x$ and $z_2 = y$ derive the flat model of the robot.

2. Using the flatness transformation find a trajectory $x(t)$, $y(t)$ to move a unicycle robot in an obstacle free environment, from an initial state $x_i = y_i = \theta_i = 0$ and $v_i = 0$ at $t_i = 0$, to a final state $x_f = y_f = 5$, $\theta_f = 0$ and $v_f = 0$ at $t_f = 1$.

3. How can the previous solution be modified in order to introduce also the minimization of the cost

$$J(v, \omega) = \int_0^{T_f} \left(v^2 + 0.1\omega^2 \right) \, \mathrm{d}t$$

where now T_f is a free parameter.

EXERCISE 4

1. Write a definition of small-time local accessibility and small-time local controllability.

2. Give an example of a robot that is small-time locally accessible and one that is small-time locally controllable, and illustrate the meaning of these properties in the context of the controllability of a mobile robot.

3. Demonstrate that the kinematic models of a unicycle and a differential drive robot are both small-time locally controllable.