Corso di Laurea Magistrale in Ingegneria Meccanica

Automatic Control A

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This file consists of **8** pages (including cover).

During the exam you are not allowed to exit the room for any other reason than handing your work or withdrawing from the exam.

You are not allowed to withdraw from the exam during the first 30 minutes.

During the exam you are not allowed to consult books or any kind of notes.

You are not allowed to use calculators with graphic display.

Solutions and answers can be given **either in English or in Italian**.

Solutions and answers must be given **exclusively in the reserved space**. Only in the case of corrections, or if the space is not sufficient, use the back of the front cover.

The clarity and the order of the answers will be considered in the evaluation.

At the end of the test you have to **hand this file only**. Every other sheet you may hand will not be taken into consideration.

Use this page ONLY in case of corrections or if the space reserved for some answers turned out to be insufficient

Exercise 1

Consider the following linear and time invariant dynamical system

$$\begin{cases} \dot{x}(t) = A\,x(t) + B\,u(t) \\ y(t) = C\,x(t) + D\,u(t) \end{cases}$$

where

$$A = \begin{bmatrix} -1 & 1 \\ 0 & \alpha \end{bmatrix} \qquad B = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \qquad C = \begin{bmatrix} 1 & 0 \end{bmatrix} \qquad D = 0$$

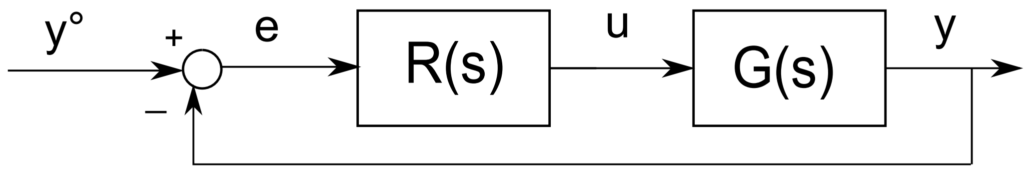
1. Compute the values of parameter $\alpha \in \mathbb{R}$ for which the system is asymptotically stable.

2. Compute the values of parameter $\alpha \in \mathbb{R}$ for which the transfer function has two time constants $\tau_1 = 1\ s$ and $\tau_2 = 2\ s$.

3. Compute the values of parameter $\alpha \in \mathbb{R}$ for which the system is completely controllable and completely observable.

Exercise 2

Consider the following control loop

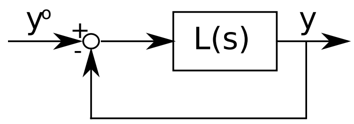


where $G(s) = \dfrac{1-s}{(1+10s)^2}$.

1. Compute the transfer function $R(s)$ of a controller in such a way that:
   - $|e_\infty| \leq 0.15$ for $y^\circ(t) = 10\text{sca}(t)$;
   - $\varphi_m \geq 65^\circ$ and ω_c is approximately maximised.

Exercise 3

Consider the following closed-loop system



where $L(s) = R(s)G(s)$, $R(s) = \dfrac{\rho}{(1+s)}$, $G(s) = \dfrac{1}{(1+s)^2}$.

1. Sketch the direct and inverse root loci.

2. Using the previous root loci, find the values of ρ for which the closed-loop system is asymptotically stable.

Exercise 4

Consider a digital control system, with transfer function $R(z)$, represented by the difference equation (from the input $e^*(k)$ to the output $u^*(k)$)

$$u^*(k) = 0.5u^*(k-1) + 0.1e^*(k-1)$$

1. Compute the transfer function $R(z)$ of the digital control system.

2. Verify that the transfer function $R(z)$ can be obtained discretising, with the Tustin method, the following continuous time control system

$$R^o(s) = 0.2\frac{1-0.25s}{1+0.75s}$$

   and determine the corresponding sampling time.

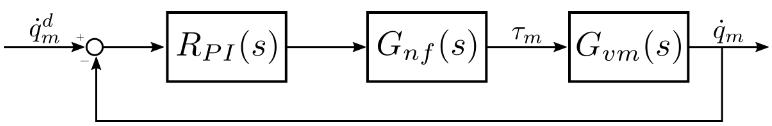
3. Determine the analytical expression of $u^*(k)$ for $e^*(k) = 0.2^k$, $k \geq 0$.

Exercise 5

1. Considering an elastic servomechanism, write the transfer function of a notch filter explaining the meaning of each parameter, and draw the magnitude Bode diagram of this transfer function.

2. Show how (draw the block diagram) the notch filter can be used to increase the damping of the closed-loop poles in a velocity control system, and explain how the notch filter parameters have to be selected.

3. Consider the following block diagram



where $G_{nf}(s)$ is a notch filter, and say if the following sentences are true or false

to tune the notch filter an accurate estimate of the motion control system natural frequency is required	**true**	**false**
the zeros of the notch filter are selected to match the pair of complex poles of the closed-loop system	**true**	**false**
the pair of low damping poles cancelled out by the notch filter appear as poles in the disturbance-output transfer function	**true**	**false**
transforming $G_{nf}(s)$ into a digital filter does not cause any distortion in the frequency response	**true**	**false**

Exercise 6

Write the input-output relations describing the following circuits