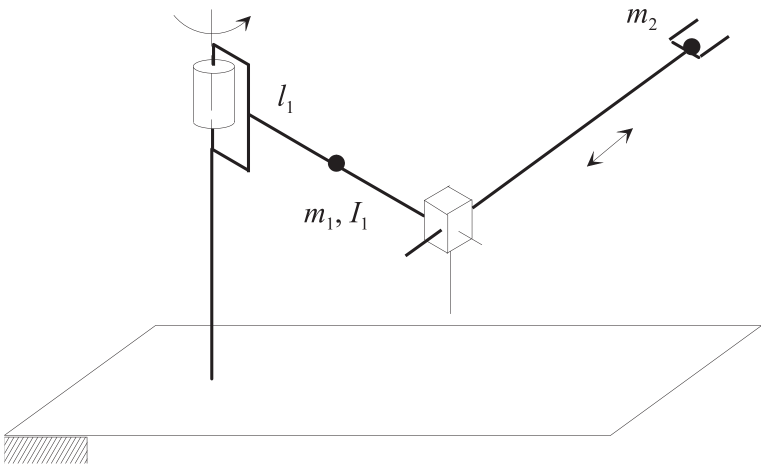
EXERCISE 1

1. Consider the manipulator sketched in the picture, where the mass of the second link is assumed to be concentrated at the end-effector:



Find the expression of the inertia matrix $\mathbf{B}(\mathbf{q})$ of the manipulator.

2. Compute the matrix $\mathbf{C}(\mathbf{q}, \dot{\mathbf{q}})$ of the Coriolis and centrifugal terms[1] for this manipulator.

3. Check that matrix $\mathbf{N}(\mathbf{q}, \dot{\mathbf{q}}) = \dot{\mathbf{B}}(\mathbf{q}) - 2\mathbf{C}(\mathbf{q}, \dot{\mathbf{q}})$, is skew symmetric.

4. Compute the expression of the derivative of the kinetic energy $\mathbf{T}(\mathbf{q}, \dot{\mathbf{q}}) = \frac{1}{2}\dot{\mathbf{q}}^T\mathbf{B}(\mathbf{q})\dot{\mathbf{q}}$ for this robot.

[1] The general expression of the Christoffel symbols is $c_{ijk} = \frac{1}{2}\left(\frac{\partial b_{ij}}{\partial q_k} + \frac{\partial b_{ik}}{\partial q_j} - \frac{\partial b_{jk}}{\partial q_i}\right)$

EXERCISE 2

Consider a kinematically redundant robot and the following general solution of the inverse kinematics at the velocity level:

$$\dot{\mathbf{q}} = \dot{\mathbf{q}}^* + \mathbf{P}\dot{\mathbf{q}}_0$$

1. Write an expression for the particular solution $\dot{\mathbf{q}}^*$ specifying the optimal problem from which this solution derives.

2. Write an expression for the projection matrix $\mathbf{P}$, proving that with such an expression the velocities $\dot{\mathbf{q}}_0$ are actually projected into the null-space of the Jacobian.

3. Explain what might be a criterion to select the velocities $\dot{\mathbf{q}}_0$ mentioning a few specific examples.

4. Sketch the block diagram of a closed loop implementation of the inverse kinematics algorithm for a redundant manipulator.

EXERCISE 3

1. The dynamic model of a mobile robot can be written, using the Lagrange equations, as

$$B(\mathbf{q})\,\ddot{\mathbf{q}} + n(\mathbf{q},\dot{\mathbf{q}}) = S(\mathbf{q})\,\boldsymbol{\tau} + A(\mathbf{q})\,\boldsymbol{\lambda}$$
$$A^T(\mathbf{q})\,\dot{\mathbf{q}} = \mathbf{0}$$

   What does each equation represent?
   What do $S(\mathbf{q})$, $\boldsymbol{\tau}$, $A(\mathbf{q})$ and $\boldsymbol{\lambda}$ represent?

2. Consider a unicycle robot. Write the expressions of matrices $B(\mathbf{q})$, $n(\mathbf{q},\dot{\mathbf{q}})$, $A(\mathbf{q})$ and $S(\mathbf{q})$.

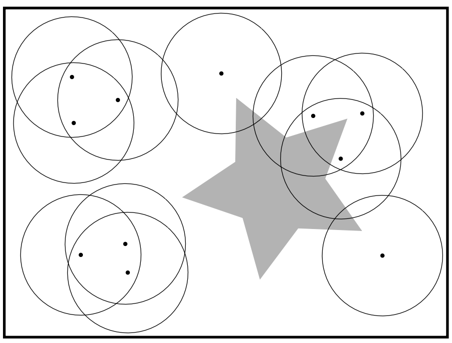
3. Write the dynamic model of a unicycle robot using the Newton-Euler formulation.

4. Write a control law that can feedback linearize the dynamic model developed in step 3.

EXERCISE 4

1. Write the algorithm of the Probabilistic RoadMap planner.

2. Given the environment and the set of sampled nodes shown in the picture (where the gray object is an obstacle, and the circles represent the neighbourhood used to compute the set of near nodes), build the roadmap using sPRM algorithm.



3. Given the environment and the set of sampled nodes shown in the previous picture, build the roadmap using PRM algorithm.

4. In many planning algorithms, especially in optimal ones, the radius used to compute the set of near nodes is not constant. A typical relation used to define a non-constant radius γ is the following

$$\gamma > \gamma^{\star} = 2\left(1 + 1/d\right)^{1/d}\left(\mu\left(\mathcal{Q}_{free}\right)/\zeta_d\right)^{1/d}$$

Explain the reason for choosing a non-constant radius.
Give an intuitive interpretation of the relation that defines $\gamma^{\star}$.