EXERCISE 1

1. A mobile robot is characterised by k kinematic constraints, that are expressed in Pfaffian form as $A^T(\mathbf{q})\,\dot{\mathbf{q}} = \mathbf{0}$, where $\mathbf{q} \in \mathbb{R}^n$ is the configuration vector. Illustrate all the steps of the procedure that allows to derive the kinematic model of the robot, and write the general expression of the kinematic model.

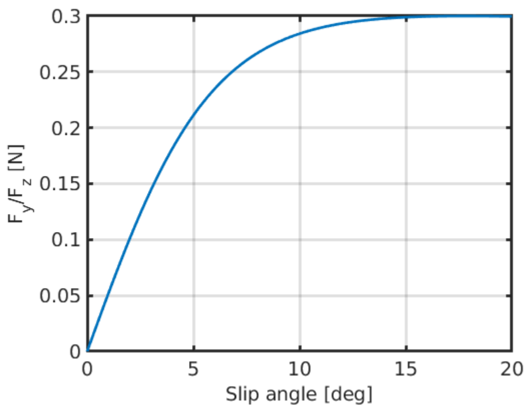
2. Perform all the steps described in the previous item, from the definition of the configuration vector and the $A(\mathbf{q})$ matrix to the expression of the kinematic model, for a unicycle robot.

3. Draw the Simulink diagram required to simulate the kinematic model of a unicycle robot. If you make use of user-defined functions include the code of each function.

EXERCISE 2

1. As a result of an experimental campaign performed on snow, a tire lateral force has been characterised interpolating experimental data with the following Pacejka Magic Formula (whose plot is shown in the picture below)

$$\frac{F_y}{F_z} = 0.3 \sin\left(2 \arctan\left(5\alpha - \left(5\alpha - \arctan\left(5\alpha\right)\right)\right)\right)$$



Determine, using the Magic Formula and motivating the result, the value of the cornering stiffness, and draw on the picture the cornering stiffness approximation of the lateral force/slip relation.

2. The cornering stiffness approximation cannot represent the tire force saturation. Illustrate a modelling approach (different from the Pacejka Magica Formula) that allows to represent the saturation and is suitable for model-based control.

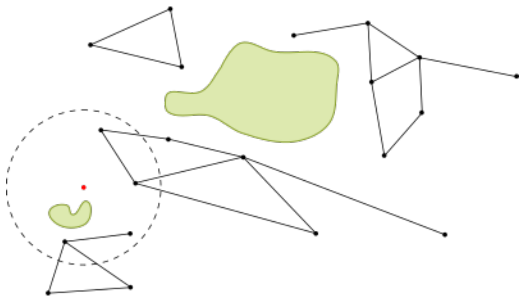
3. During a curve the same tire is characterised by a slip angle of 5 deg. What is the corresponding value of the lateral force F_y, assuming $F_z = 150$ N? What is the maximum longitudinal force F_x the tire can generate in these conditions?

EXERCISE 3

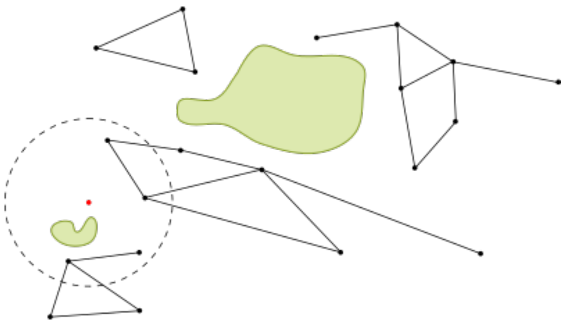
1. Write and explain the pseudocode of the algorithm to construct the probabilistic roadmap used by PRM.

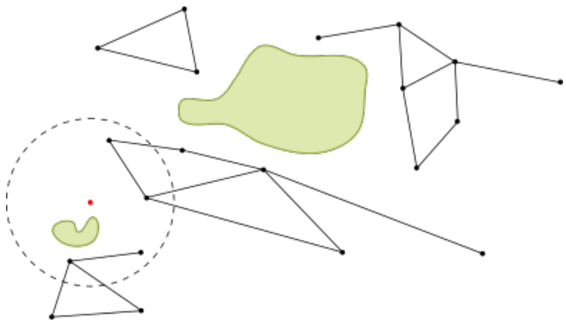
2. Explain how the previous algorithm has to be modified in order to obtain sPRM and PRM* algorithms.

3. After the execution of some iterations of PRM algorithm we have the situation depicted below



where the black dots are the nodes sampled in previous iterations, the red dot is the node sampled in the actual iteration, the green blobs are obstacles and the black dashed circle is the neighbourhood considered to defined the Near set.
Draw in the left picture the result of an iteration of PRM and in the right picture the result of an iteration of sPRM.





EXERCISE 4

1. What is the *canonical simplified model for nonholonomic mobile robots*? Why is it important in the context of designing a controller for a nonholonomic robot?

2. Show how the unicycle, differential drive, and rear-wheel drive bicycle kinematic models can be made equivalent to the canonical model.

3. Consider a bicycle kinematic model without reverse. Show how the actuation constraints $0 \leq v \leq v_M$ and $-\phi_M \leq \phi \leq \phi_M$ can be rewritten in terms of the canonical model input variables.