EXERCISE 1

1. Explain the difference between the following types of kinematic constraints:
   - bilateral / unilateral;
   - rheonomical / scleronomical;
   - holonomic / nonholonomic.

2. $\dot{\mathbf{q}} = G(\mathbf{q})\,\mathbf{u}$ is the kinematic model of a mobile robot. Clearly explain what $\mathbf{q}$ and $G(\mathbf{q})$ represent. How is $G(\mathbf{q})$ computed from the kinematic constraints?

3. Write the kinematic model of a differential drive robot considering as input the rotational velocities ω_R and ω_L of the left and right wheels.

EXERCISE 2

1. The dynamic model of a mobile robot can be written, using the Lagrange equations, as

$$B(\mathbf{q})\ddot{\mathbf{q}} + n(\mathbf{q}, \dot{\mathbf{q}}) = S(\mathbf{q})\boldsymbol{\tau} + A(\mathbf{q})\boldsymbol{\lambda}$$
$$A^T(\mathbf{q})\dot{\mathbf{q}} = \mathbf{0}$$

   What does each equation represent?
   What do $S(\mathbf{q})$, $\boldsymbol{\tau}$, $A(\mathbf{q})$ and $\boldsymbol{\lambda}$ represent?

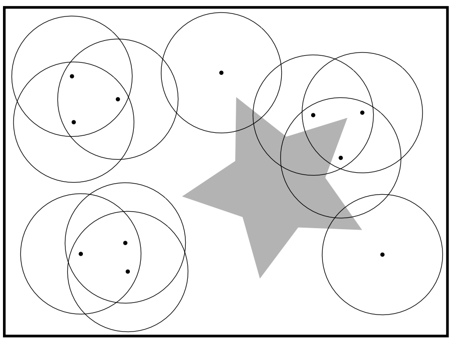
2. Consider a unicycle robot. Write the expressions of matrices $B(\mathbf{q})$, $n(\mathbf{q}, \dot{\mathbf{q}})$, $A(\mathbf{q})$ and $S(\mathbf{q})$.

3. Write the dynamic model of a unicycle robot using the Newton-Euler formulation.

ESERCIZIO 3

1. Write the algorithm of the Probabilistic RoadMap planner.

2. Given the environment and the set of sampled nodes shown in the picture (where the gray object is an obstacle, and the circles represent the neighbourhood used to compute the set of near nodes), build the roadmap using sPRM algorithm.



3. In many planning algorithms, especially in optimal ones, the radius used to compute the set of near nodes is not constant. A typical relation used to define a non-constant radius r is the following

$$r(V) = \gamma \left(\frac{\log\left(\mathrm{card}\left(V\right)\right)}{\mathrm{card}\left(V\right)} \right)^{1/d}$$

Explain the reason for choosing a non-constant radius.
Give an intuitive interpretation of the relation that defines $r(V)$.

ESERCIZIO 4

1. Consider a unicycle kinematic model. Assuming $z_1 = x$ and $z_2 = y$ as flat outputs, write the expression of the state and input variables as functions of the flat outputs and their derivatives.

2. How can the previous relations (flat representation of the unicycle kinematic model) be used to set up a trajectory tracking controller? Show a block diagram of the control system, including trajectory generation, controller and robot model.
   For each block specify the equations relating the inputs to the outputs. For trajectory generation consider a circle in the xy plane.

3. Illustrate the main pros and cons of the previous control solution with respect to a trajectory tracking controller based on feedback linearization.