Control of Mobile Robots

Prof. Bascetta

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NAME:
UNIVERSITY ID NUMBER:

SIGNATURE: ________________________________

Warnings

- This file consists of **8** pages (including cover).
- During the exam you are not allowed to exit the room for any other reason than handing your work or withdrawing from the exam.
- You are not allowed to withdraw from the exam during the first 30 minutes.
- During the exam you are not allowed to consult books or any kind of notes.
- You are not allowed to use calculators with graphic display.
- Solutions and answers can be given **either in English or in Italian**.
- Solutions and answers must be given **exclusively in the reserved space**. Only in the case of corrections, or if the space is not sufficient, use the back of the front cover.
- The clarity and the order of the answers will be considered in the evaluation.
- At the end of the test you have to **hand this file only**. Every other sheet you may hand will not be taken into consideration.

EXERCISE 1

1. Consider a wheel rolling without slipping on the horizontal plane, keeping the sagittal plane in the vertical direction. Write the expression of the pure rolling constraint in the case of a fixed and a steerable wheel.

2. Are the previous constraints holonomic or nonholonomic? Use the necessary and sufficient condition to support your answer.

3. Consider a single-track robot with front fixed wheel and rear steerable wheel. Assuming as configuration vector $\mathbf{q} = [x\ y\ \theta\ \phi]^T$, where $(x\ y\ \theta)$ is the robot pose and ϕ the steering angle, write the kinematic constraints in Pfaffian form that allow to derive the kinematic model of the robot.

EXERCISE 2

Consider a rear-wheel drive bicycle robot under the following assumptions:

- braking force can be neglected;
- sideslip and steering angles are small (i.e., $\cos x \approx 1$ and $\sin x \approx x$);
- force-slip relation is linear;
- the absolute velocity is slowly varying.

1. Write the configuration vector and the equations describing the kinematic model. Explain the meaning of each symbol used in the equations.

2. Write the equations describing the dynamic model considering the previous assumptions and using as state variables sideslip and yaw rate. Explain the meaning of each symbol used in the equations.

3. Consider now a force-slip relation for the lateral force that is linear up to the maximum available force (given by friction), and then saturates to the friction force. Write the expression of the front/rear lateral forces and the equations describing the dynamic model of the vehicle, based on this force-slip relation. Explain the meaning of each symbol used in the equations.

EXERCISE 3

1. Consider a unicycle mobile robot. Selecting as flat outputs $z_1 = x$ and $z_2 = y$ derive the flat model of the robot.

2. Using the flatness transformation find a trajectory $x(t)$, $y(t)$ to move a unicycle robot in an obstacle free environment, from an initial state $x_i = y_i = \theta_i = 0$ and $v_i = 0$ at $t_i = 0$, to a final state $x_f = y_f = 5$, $\theta_f = 0$ and $v_f = 0$ at $t_f = 1$.

3. How can the previous solution be modified in order to introduce also the minimization of the cost

$$J(v, \omega) = \int_0^{T_f} \left(v^2 + 0.1\omega^2\right) \, \mathrm{d}t$$

where now T_f is a free parameter. Write all the steps of the algorithm that allows to plan the $x(t)$, $y(t)$ robot trajectory.

EXERCISE 4

1. What is the *canonical simplified model for nonholonomic mobile robots*? Why is it important in the context of designing a controller for a nonholonomic robot?

2. Show how the unicycle, differential drive, and rear-wheel drive bicycle kinematic models can be made equivalent to the canonical model.

3. Consider a bicycle kinematic model without reverse. Show how the actuation constraints $0 \leq v \leq v_M$ and $-\phi_M \leq \phi \leq \phi_M$ can be rewritten in terms of the canonical model input variables.