Control of Industrial and Mobile Robots

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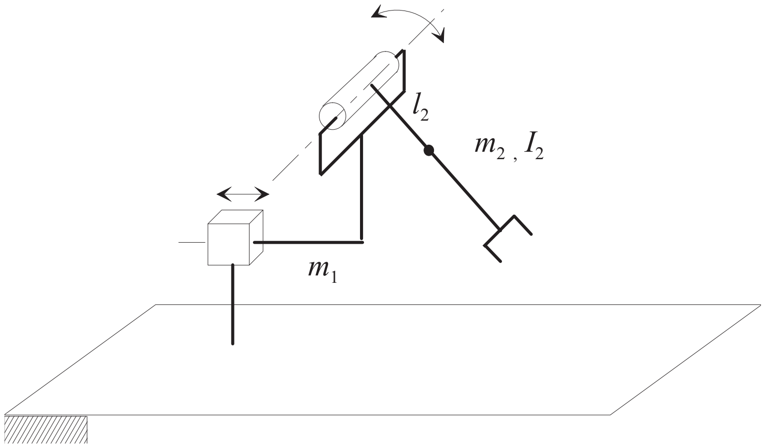
Warnings

- This file consists of **8** pages (including cover).
- During the exam you are not allowed to exit the room for any other reason than handing your work or withdrawing from the exam.
- You are not allowed to withdraw from the exam during the first 30 minutes.
- During the exam you are not allowed to consult books or any kind of notes.
- You are not allowed to use calculators with graphic display.
- Solutions and answers can be given **either in English or in Italian**.
- Solutions and answers must be given **exclusively in the reserved space**. Only in the case of corrections, or if the space is not sufficient, use the back of the front cover.
- The clarity and the order of the answers will be considered in the evaluation.
- At the end of the test you have to **hand this file only**. Every other sheet you may hand will not be taken into consideration.

EXERCISE 1

1. Consider the manipulator sketched in the picture:



Find the expression of the inertia matrix $\mathbf{B}(\mathbf{q})$ of the manipulator[1].

[1] The cross product between vector $a = \begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix}$ and $b = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$ is $c = a \times b = \begin{bmatrix} a_2 b_3 - a_3 b_2 \\ a_3 b_1 - a_1 b_3 \\ a_1 b_2 - a_2 b_1 \end{bmatrix}$

2. Compute the matrix $\mathbf{C}(\mathbf{q}, \dot{\mathbf{q}})$ of the Coriolis and centrifugal terms[2] for this manipulator.

3. Ignoring the gravitational terms, compute the dynamic model of this manipulator.

4. Write the dynamic model in a linear form with respect to a set of dynamic parameters.

[2]The general expression of the Christoffel symbols is $c_{ijk} = \frac{1}{2}\left(\frac{\partial b_{ij}}{\partial q_k} + \frac{\partial b_{ik}}{\partial q_j} - \frac{\partial b_{jk}}{\partial q_i}\right)$

EXERCISE 2

1. Sketch the block diagram of a position/speed control of a servomechanism, including a speed feed-forward action.

2. Consider now the speed control system. Write the expression of the loop transfer function $L_v(s)$, draw its amplitude Bode diagram and discuss how the speed controller can be tuned based on such Bode diagram.

3. Assume now the following values for the physical parameters of the servomechanism.

$$\begin{aligned} J_m &= 0.02 \ kgm^2 \\ D_m &\approx 0 \\ \rho &= 2 \end{aligned}$$

where ρ is the inertia ratio. Design a speed PI controller in such a way to obtain a crossover frequency $\omega_{cv} = 100$ rad/s.

4. Explain what is the difference between the repeatability and the accuracy of an industrial robot.

EXERCISE 3

1. Given a kinematic constraint in Pfaffian form

$$X(\mathbf{q})\,\dot{x} + Y(\mathbf{q})\,\dot{y} + Z(\mathbf{q})\,\dot{z} = 0$$

where $\mathbf{q} = \begin{bmatrix} x & y & z \end{bmatrix}$ is the configuration vector. Illustrate the necessary and sufficient condition for this constraint to be holonomic.

2. Consider the following kinematic constraints

$$\dot{q}_1 + q_1\dot{q}_2 + \dot{q}_3 = 0 \qquad \dot{q}_1 + \dot{q}_2 + q_1\dot{q}_3 = 0$$

where $\mathbf{q} \in \mathbb{R}^3$ is the configuration vector. Verify, using a formal demonstration, if each of these constraints by itself is holonomic or nonholonomic.

3. Consider the system composed by two constraints in item 2. Does the holonomicity/nonholonomicity of each constraint by itself imply the holonomicity/nonholonomicity of the system of constraints? Is the system of two constraints integrable?

4. Considering the first constraint ($a^T(\mathbf{q})\dot{\mathbf{q}} = \dot{q}_1 + q_1\dot{q}_2 + \dot{q}_3 = 0$), two vectors in the null of $a^T(\mathbf{q})$ are

$$\mathbf{g}_1 = \begin{bmatrix} q_1 \\ -1 \\ 0 \end{bmatrix} \qquad \mathbf{g}_2 = \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$$

Compute the vector field representing the motion that is locally constrained by $\dot{q}_1 + q_1\dot{q}_2 + \dot{q}_3 = 0$.

EXERCISE 4

1. Consider a unicycle robot carrying a box full of sand, centred with respect to the robot center of gravity. The robot mass and yaw moment of inertia are equal to 30 Kg and 0.8 Kgm^2, when the box is empty, and 60 Kg and 1.6 Kgm^2, when the box is full. During the motion the box is leaking sand, and the mass and yaw moment of inertia change with time according to the following relations

$$M(t) = 60 - 0.5t \qquad I(t) = \frac{8}{5} - \frac{t}{75}$$

Write the dynamic model of the robot, assuming that there is no relative motion between the sand and the box and that the change of mass is slow compared to the motion of the robot.

2. The robot tire lateral force-slip relations are modelled using a piecewise constant model

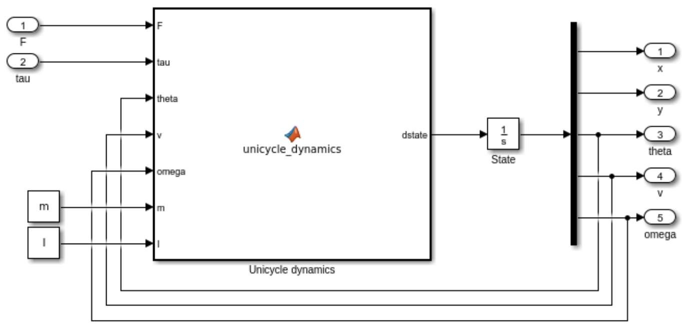
$$F_y = \begin{cases} C_\alpha \alpha & \alpha \leq \alpha_{lin} \\ F_{y_{max}} & \alpha > \alpha_{lin} \end{cases}$$

where C_α is the cornering stiffness, α the slip angle, and $\alpha_{lin} = 0.1$ rad. The friction coefficient is $\mu = 0.5$.
Write the lateral force-slip relation at $t_1 = 0$ and $t_2 = 30$.

3. Consider that the robot is equipped with wheel velocity controllers tuned for the case of empty box. For the design of a model-based trajectory tracking controller, assuming either the linear and angular velocity or the wheel torque are available to the controller, is it better to consider the kinematic

or dynamic model of the robot? Clearly motivate the answer.

4. The following Simulink diagram is used to simulate the model of a unicycle robot



Write the code used by the user-defined Matlab function that is inside the block "Unicycle dynamics".