# Control of Industrial and Mobile Robots

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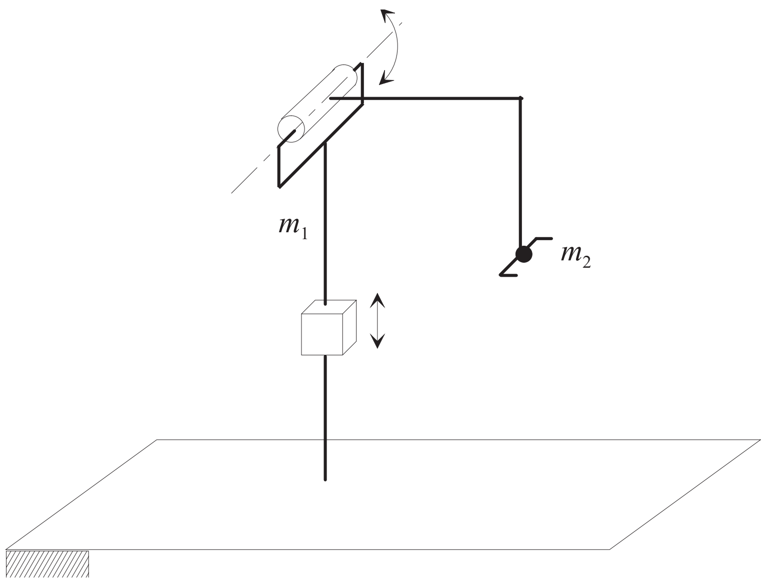
**Warnings**

- This file consists of **10** pages (including cover).
- During the exam you are not allowed to exit the room for any other reason than handing your work or withdrawing from the exam.
- You are not allowed to withdraw from the exam during the first 90 minutes.
- During the exam you are not allowed to consult books or any kind of notes.
- You are not allowed to use calculators with graphic display.
- Solutions and answers can be given **either in English or in Italian**.
- Solutions and answers must be given **exclusively in the reserved space**. Only in the case of corrections, or if the space is not sufficient, use the back of the front cover.
- The clarity and the order of the answers will be considered in the evaluation.
- At the end of the test you have to **hand this file only**. Every other sheet you may hand will not be taken into consideration.

**EXERCISE 1**

1. Consider the manipulator sketched in the picture, where the mass of the second link is assumed to be concentrated at the end-effector:




Find the expression of the inertia matrix $\mathbf{B}(\mathbf{q})$ of the manipulator[1].

[1]The cross product between vector $a = \begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix}$ and $b = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$ is $c = a \times b = \begin{bmatrix} a_2 b_3 - a_3 b_2 \\ a_3 b_1 - a_1 b_3 \\ a_1 b_2 - a_2 b_1 \end{bmatrix}$

2. Compute the matrix $\mathbf{C}(\mathbf{q}, \dot{\mathbf{q}})$ of the Coriolis and centrifugal terms[2] for this manipulator.

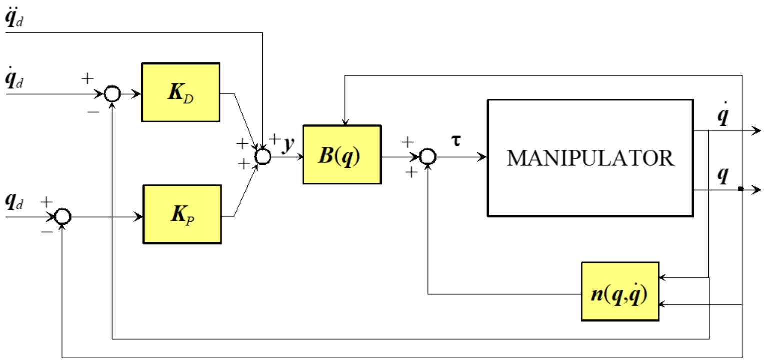
3. Ignoring the gravitational terms, write the dynamic model for this manipulator.

[2]The general expression of the Christoffel symbols is $c_{ijk} = \frac{1}{2}\left(\frac{\partial b_{ij}}{\partial q_k} + \frac{\partial b_{ik}}{\partial q_j} - \frac{\partial b_{jk}}{\partial q_i}\right)$

4. Show that the model obtained in the previous step is linear with respect to a set of dynamic parameters.

**EXERCISE 2**

1. Consider the control scheme sketched in the following picture:




Explain which control scheme it refers to and what is the result in terms of closed-loop dynamics that can be achieved with such control scheme.

2. In a two-link planar manipulator in the vertical plane with prismatic joints, the inertia matrix and the gravitational terms take the following expressions, respectively:

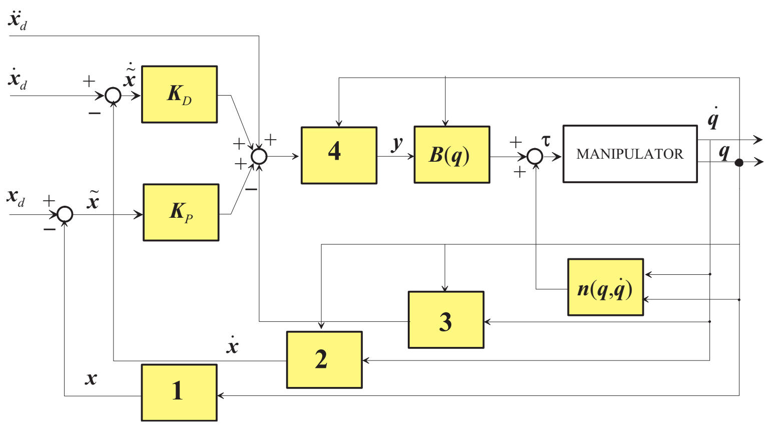
$$\mathbf{B} = \begin{bmatrix} m_1 + m_2 & 0 \\ 0 & m_2 \end{bmatrix}$$

$$\mathbf{g} = \begin{bmatrix} (m_1 + m_2)\, g \\ 0 \end{bmatrix}$$

Write the expression (equation by equation) of the control law for the control scheme of this exercise, specific for this manipulator.

3. Tune the two matrices $\mathbf{K}_P$ and $\mathbf{K}_D$ in such a way that the dynamics of the error in the two joints is identical with two real eigenvalues at frequency 10 rad/s

4. In the operational space version of the control scheme of this exercise, four more blocks appear, numbered as 1, 2, 3, 4 in the following sketch:




Without adding any further comment, write the mathematical expressions of the blocks 1, 2, 3, 4.

**EXERCISE 3**

1. Given the kinematic constraint

$$\dot{q}_1 - q_1\dot{q}_2 + 4\dot{q}_3 = 0$$

   where $\mathbf{q} = \begin{bmatrix} q_1 & q_2 & q_3 \end{bmatrix}$ is the configuration vector. Determine, using the necessary and sufficient condition, if this constraint is holonomic or nonholonomic.

2. Given the kinematic constraint

$$2\dot{q}_2 - q_1\dot{q}_3 = 0$$

   where $\mathbf{q} = \begin{bmatrix} q_1 & q_2 & q_3 \end{bmatrix}$ is the configuration vector. Determine, using the necessary and sufficient condition, if this constraint is holonomic or nonholonomic.

3. Is the system of two constraints

$$\dot{q}_1 - q_1\dot{q}_2 + 4\dot{q}_3 = 0 \qquad 2\dot{q}_2 - q_1\dot{q}_3 = 0$$

holonomic or nonholonomic? Motivate the answer analysing the accessibility distribution.

4. Considering the two constraints separately, are these

$$\begin{aligned} \dot{q}_1 &= q_1 u_1 - 4u_2 \\ \dot{q}_2 &= u_1 \\ \dot{q}_3 &= u_2 \end{aligned} \qquad\qquad \begin{aligned} \dot{q}_1 &= u_2 \\ \dot{q}_2 &= 0.5 q_1 u_1 \\ \dot{q}_3 &= u_1 \end{aligned}$$

the kinematic models associated to the constraints in 1 and 2? Clearly motivate the answer.

**EXERCISE 4**

Consider a simplified version of the rear-wheel drive bicycle model

$$\dot{x} = v \cos \theta$$
$$\dot{y} = v \sin \theta$$
$$\dot{\theta} = \frac{v}{\ell} \tan \phi$$

where $(x, y, \theta)$ is the position and orientation of the vehicle, $v$ the linear velocity, and $\phi$ the steering angle.

1. Write the expression of the feedback linearising law for this model.

2. Is the previous linearising feedback affected by any singularity? Does it introduce any hidden dynamics? If yes, which are the states that belong to the hidden dynamics? Clearly motivate the answer.

3. Write the equations of the dynamical system representing the closed-loop system obtained connecting the model with the controller.

4. Assuming as inputs

$$v_{xP}(t) = \bar{v}_P \cos \bar{\theta}_P \qquad v_{yP}(t) = \bar{v}_P \sin \bar{\theta}_P$$

with $\bar{v}_P$ and $\bar{\theta}_P$ constants, and $\bar{v}_P > 0$.
Study the stability of the hidden state.