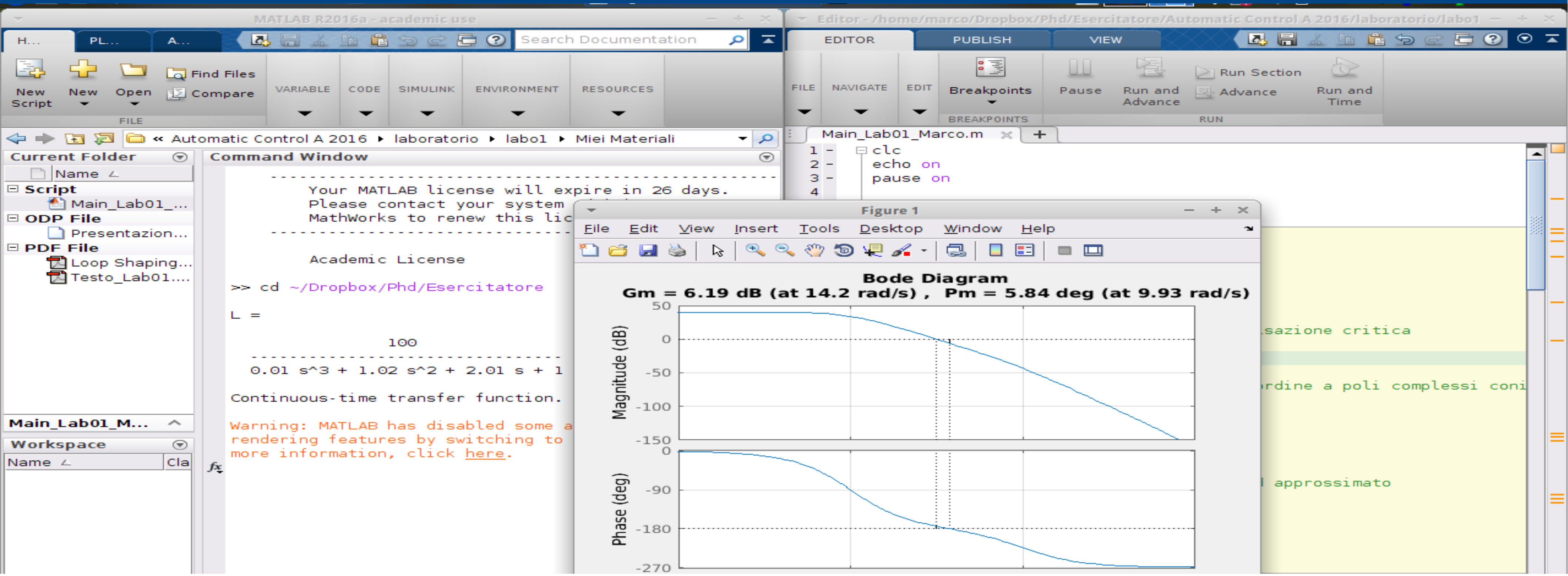
Automatic Control

A.Y. 2017-18, Master of Science degree in Mechanical Engineering

Prof. L. Bascetta



Laboratory 2: "Pole Placement"

7th November 2017

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Laboratories Schedule

- Lab 1: “**Closed Loop Systems**”
  24/10/17

- Lab 2: “**Pole Placement**”
  7/11/17

- Lab 3: “**Discrete time and digital control systems**”
  21/11/17

- Lab 4: “**Motion control -standard control techniques**”
  28/11/17

- Lab 5: “**Motion Control – advanced control techniques**”
  5/12/17

Lab CS.02

15:15 – 16:45

Needed Software

Mathworks Matlab®:

- **Control System Toolbox**
- **Simulink**

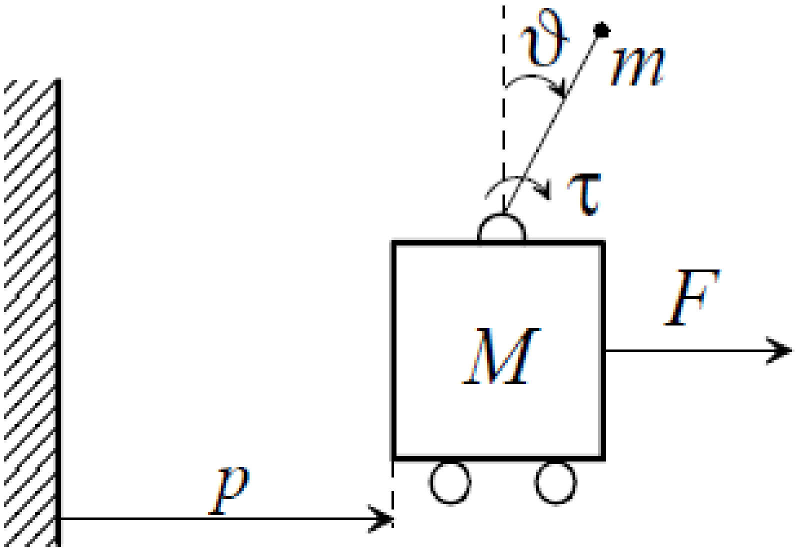
Type `ver` in the Matlab Command Window

A simple advice

One of the best features of Matlab is its support documentation ...

When you don't know either what a command does or the syntax of the command, type
`help` *`command name`*
or **`doc` *`command name`*** for a more comprehensive explanation
(Ex: `help bode`, `doc damp`)

Dynamic Model Linearization

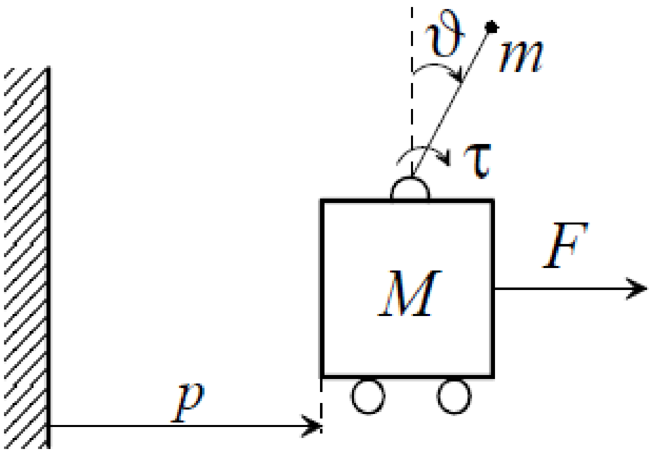


- F,τ = control inputs (**u**)
- p,θ = generalized coordinates
- M,m,l = system parameters

$$\begin{cases} (M+m)\ddot{p} - ml\dot{\theta}^2 sin(\theta) + ml\ddot{\theta}cos(\theta) = F \\ ml^2\ddot{\theta} + ml\ddot{p}cos(\theta) - mglsin(\theta) = \tau \end{cases}$$

NONLINEAR Equations of Motions
(= Nonlinear Dynamic Model)

Dynamic Model Linearization



For F = τ = 0 the equilibrium point is:

$$\theta_{eq} = \dot{\theta}_{eq} = 0$$

$$p_{eq} = \dot{p}_{eq} = 0$$

$$\begin{cases} \delta\ddot{p} &= -\dfrac{m}{M} g\delta\theta + \dfrac{1}{M}\delta F - \dfrac{1}{lM}\delta\tau \\ \\ \delta\ddot{\theta} &= \dfrac{g}{l}\dfrac{M+m}{M}\delta\theta - \dfrac{1}{lM}\delta F + \dfrac{1}{l^2}\dfrac{M+m}{Mm}\delta\tau \end{cases}$$

$$\delta p = p - p_{eq} \qquad \delta\theta = \theta - \theta_{eq}$$

$$\dot{\delta p} = \dot{p} \qquad \dot{\delta\theta} = \dot{\theta}$$

$$\ddot{\delta p} = \ddot{p} \qquad \ddot{\delta\theta} = \ddot{\theta}$$

Exercise 1

State-Space Representation

$$\begin{cases} \dot{\mathbf{x}} &= A\mathbf{x} + B\mathbf{u} \\ \mathbf{y} &= C\mathbf{x} + D\mathbf{u} \end{cases}$$

$$\mathbf{x} = \begin{bmatrix} x_1 = \delta p \\ x_2 = \delta \dot{p} \\ x_3 = \delta\theta \\ x_4 = \delta\dot{\theta} \end{bmatrix} \in \mathbb{R}^{n=4}$$

$$\mathbf{u} = \begin{bmatrix} u_1 &= \delta F \\ u_2 &= \delta\tau \end{bmatrix} \in \mathbf{R}^{p=2}$$

$$\mathbf{y} = \begin{bmatrix} y_1 &= \delta p \\ y_2 &= \delta\theta \end{bmatrix} \in \mathbf{R}^{m=2}$$

$$\frac{d}{dt}\delta p = \delta\dot{p}$$

$$\frac{d}{dt}\delta\dot{p} = -\frac{m}{M}g\,\delta\theta + \frac{1}{M}\delta F - \frac{1}{lM}\delta\tau$$

$$\frac{d}{dt}\delta\theta = \delta\dot{\theta}$$

$$\frac{d}{dt}\delta\dot{\theta} = \frac{g}{l}\,\frac{M+m}{M}\delta\theta - \frac{1}{lM}\delta F + \frac{1}{l^2}\,\frac{M+m}{Mm}\delta\tau$$

State-Space Representation

$$\begin{cases} \dot{\mathbf{x}} &= A\mathbf{x} + B\mathbf{u} \\ \mathbf{y} &= C\mathbf{x} + D\mathbf{u} \end{cases} \qquad \mathbf{x} = \begin{bmatrix} x_1 = \delta p \\ x_2 = \delta \dot{p} \\ x_3 = \delta \theta \\ x_4 = \delta \dot{\theta} \end{bmatrix} \qquad \mathbf{u} = \begin{bmatrix} u_1 = \delta F \\ u_2 = \delta \tau \end{bmatrix} \qquad \mathbf{y} = \begin{bmatrix} y_1 = \delta p \\ y_2 = \delta \theta \end{bmatrix}$$

$$A = \begin{array}{c} \\ \delta\dot{p} \\ \delta\ddot{p} \\ \delta\dot{\theta} \\ \delta\ddot{\theta} \end{array} \begin{array}{c} \begin{array}{cccc} \delta p & \delta\dot{p} & \delta\theta & \delta\dot{\theta} \end{array} \\ \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & -\frac{m}{M}g & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & \frac{g}{l}\frac{M+m}{M} & 0 \end{bmatrix} \end{array}$$

$$B = \begin{array}{c} \\ \delta\dot{p} \\ \delta\ddot{p} \\ \delta\dot{\theta} \\ \delta\ddot{\theta} \end{array} \begin{array}{c} \begin{array}{cc} \delta F & \delta\tau \end{array} \\ \begin{bmatrix} 0 & 0 \\ \frac{1}{M} & -\frac{1}{lM} \\ 0 & 0 \\ -\frac{1}{lM} & \frac{1}{l^2}\frac{M+m}{Mm} \end{bmatrix} \end{array}$$

$$C = \begin{array}{c} \\ \delta p \\ \delta\theta \end{array} \begin{array}{c} \begin{array}{cccc} \delta p & \delta\dot{p} & \delta\theta & \delta\dot{\theta} \end{array} \\ \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \end{array}$$

$$D = \begin{array}{c} \\ \delta p \\ \delta\theta \end{array} \begin{array}{c} \begin{array}{cc} \delta F & \delta\tau \end{array} \\ \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \end{array}$$

Exercise 2

Controllability and Observability

Controllability

$$CO = \left[B, AB, A^2B, \ldots, A^{n-1}B\right] \in \mathbb{R}^{n \times np}$$

Sys is Controllable if *rank (CO)*= *n* (CO full rank)

Observability

$$O = \begin{bmatrix} C \\ CA \\ CA^2 \\ \ldots \\ CA^{n-1} \end{bmatrix} \in R^{nm \times n}$$

Sys is Observable if *rank (O)*= *mn* (O full rank)

1) δF as input, δp as output

$$A_1 = A \in \mathbb{R}^{4\times 4}$$

$$B_1 = \delta F \in \mathbb{R}^{1}$$

$$C_1 = \begin{bmatrix} 1 & 0 & 0 & 0 \end{bmatrix} \in \mathbb{R}^{1\times 4}$$

$$CO_1 \in \mathbb{R}^{4\times 4}$$

$$O_1 \in \mathbb{R}^{4\times 4}$$

2) δtau as input, δθ as output

$$A_2 = A \in \mathbb{R}^{4\times 4}$$

$$B_2 = \delta\tau \in \mathbb{R}^{1}$$

$$C_2 = \begin{bmatrix} 0 & 0 & 1 & 0 \end{bmatrix} \in \mathbb{R}^{1\times 4}$$

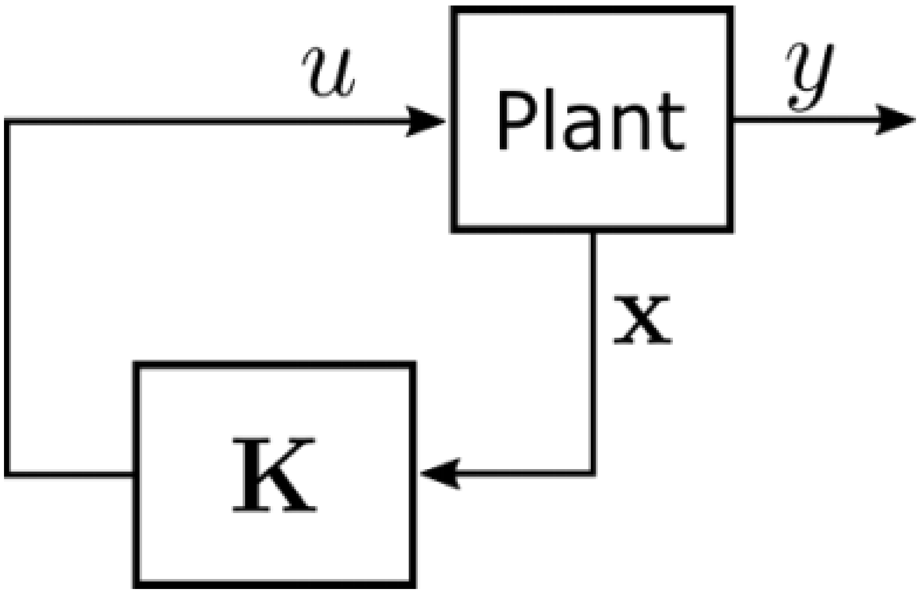
$$\Downarrow$$

$$CO_2 \in \mathbb{R}^{4\times 4}$$

$$O_2 \in \mathbb{R}^{4\times 4}$$

Exercise 3

Pole placement by full-state feedback

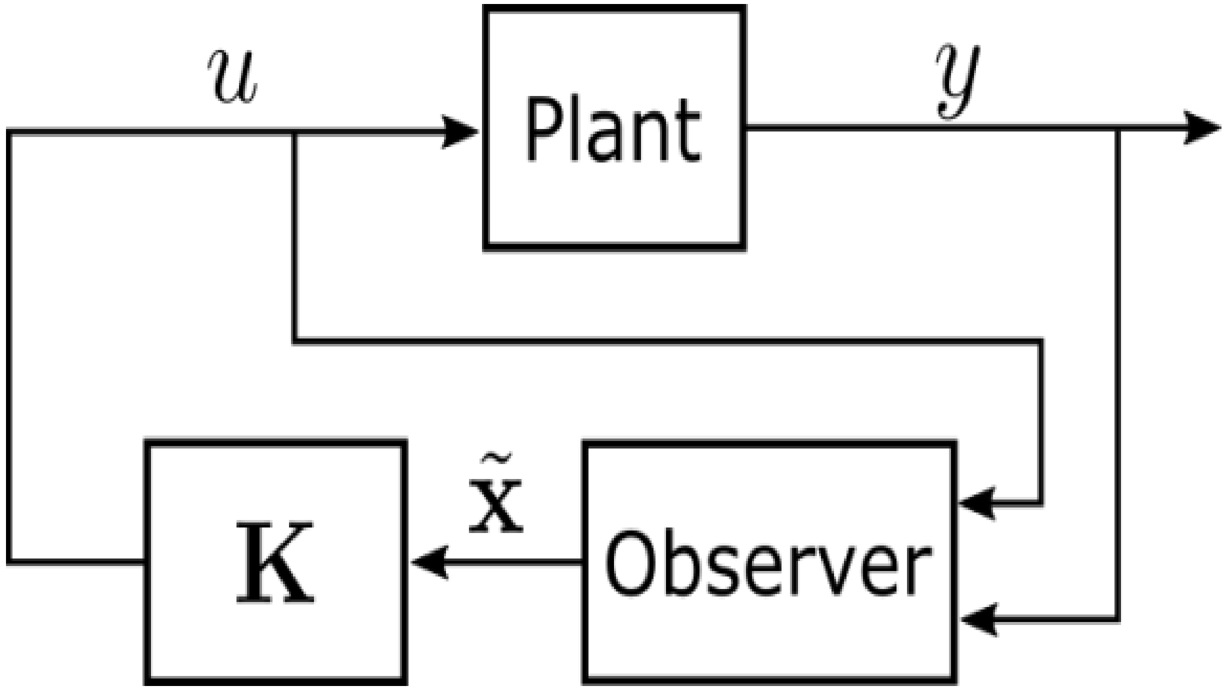


$$u = [K]\mathbf{x}$$

$$\dot{\mathbf{x}}(t) = (A + BK)\,\mathbf{x}(t)$$

Exercise 4

Full-state Observer



Observer = “Software” Sensor

Full-state Observer Design

$$\dot{\mathbf{e}} = \dot{\mathbf{x}} - \dot{\tilde{\mathbf{x}}} = (A + LC)\mathbf{e}$$

We want to place the poles of this matrix

We want to solve the same problem as before i.e. when we have placed the poles of (A+BK)...

$$\lambda_i\left[\mathbf{A} + \mathbf{LC}\right] = \lambda_i\left[(\mathbf{A} + \mathbf{LC})^T\right] = \lambda_i\left[\mathbf{A}^T + \mathbf{C}^T\mathbf{L}^T\right]$$

A + B K

```
K=-place(A,B,p)
    L=-place(A',C',pobs)'
```

Exercise 5

What's Simulink?

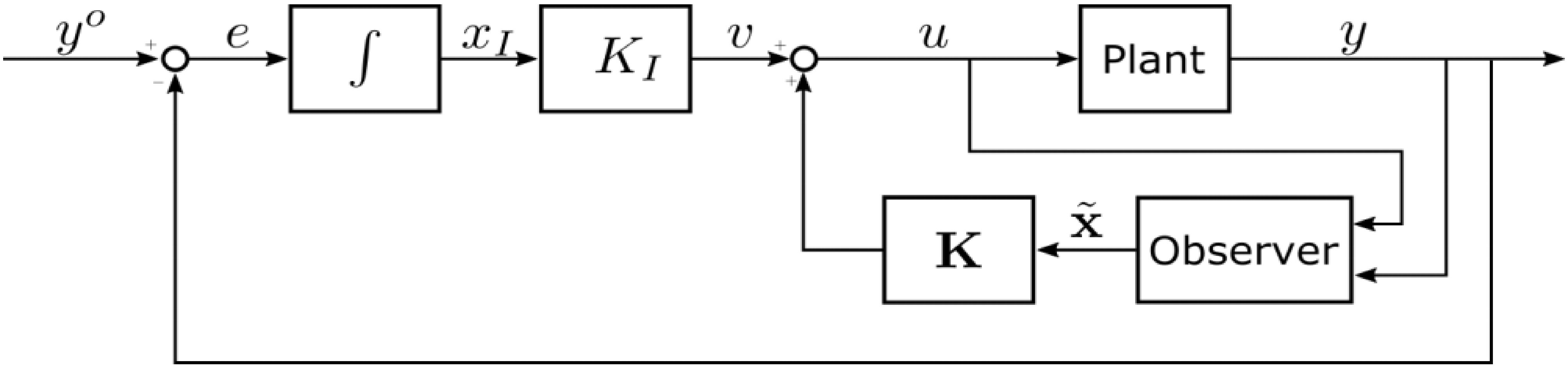
"Simulink, developed by MathWorks, is a ***graphical*** *programming environment for modeling, simulating and analyzing multidomain dynamic systems"* [Wiki]

Type `simulink` in the Matlab Command window

Simulink can use variable loaded in the matlab workspace and it can also write data in variables stored in the matlab workspace

Exercise 6

Tracking of a reference signal



$$\begin{cases} \dot{\mathbf{x}}(t) & = A\mathbf{x} + Bu \\ \dot{x_I}(t) & = e(t) = y^0(t) - y(t) = y^0(t) - C\mathbf{x}(t) \end{cases}$$

$$\begin{bmatrix} \dot{\mathbf{x}}(t) \\ \dot{x}_I(t) \end{bmatrix} = \underbrace{\begin{bmatrix} \mathbf{A} & \mathbf{0} \\ -\mathbf{C} & 0 \end{bmatrix}}_{\mathbf{F}} \begin{bmatrix} \mathbf{x}(t) \\ x_I(t) \end{bmatrix} + \underbrace{\begin{bmatrix} \mathbf{B} \\ 0 \end{bmatrix}}_{\mathbf{G}_u} u(t) + \underbrace{\begin{bmatrix} 0 \\ 1 \end{bmatrix}}_{\mathbf{G}_{y^o}} y^o(t)$$

(F,G_u) is completely controllable iff the sys has no zero in the origin s=0