Corso di Laurea Magistrale in Ingegneria Meccanica

Automatic Control A

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POLITECNICO MILANO 1863

Name: ______________________________

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University ID number: ______________________________

Signature: ______________________________

This file consists of **8** pages (including cover).

During the exam you are not allowed to exit the room for any other reason than handing your work or withdrawing from the exam.

You are not allowed to withdraw from the exam during the first 30 minutes.

During the exam you are not allowed to consult books or any kind of notes.

You are not allowed to use calculators with graphic display.

Solutions and answers can be given **either in English or in Italian**.

Solutions and answers must be given **exclusively in the reserved space**. Only in the case of corrections, or if the space is not sufficient, use the back of the front cover.

The clarity and the order of the answers will be considered in the evaluation.

At the end of the test you have to **hand this file only**. Every other sheet you may hand will not be taken into consideration.

Use this page ONLY in case of corrections or if the space reserved for some answers turned out to be insufficient

Exercise 1

Consider the following time invariant dynamical system

$$\begin{cases} \dot{x}_1 = -2x_1 + e^2 \\ \dot{x}_2 = -x_2 + u \\ y = x_2 \end{cases}$$

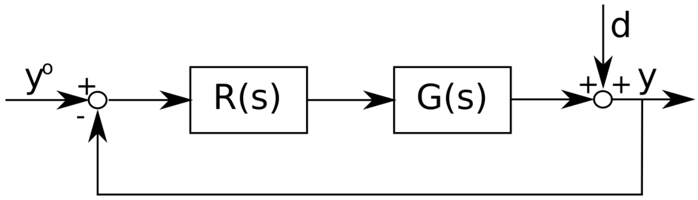
1. Compute the state and output equilibrium corresponding to $u(t) = \bar{u}$.

2. Perform stability analysis. Is stability a property of the equilibrium point or of the system? Clearly motivate the answer.

3. Write the expression of the state and output trajectory generated by the initial state $x(0) = [0 \;\; 1]$ and by the input $u(t) = \text{sca}(t)$.

Exercise 2

Consider the following control loop

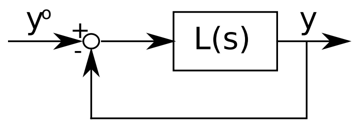


where $G(s) = 10\dfrac{1-s}{1+10s}$.

1. Compute the transfer function $R(s)$ of a controller, with order less or equal to 4, in such a way that:
   - $|e_\infty| = 0$ for $y^\circ(t) = A\text{sca}(t)$, where A is an arbitrary real constant, and $d(t) = 0$;
   - a disturbance $d(t) = Dsin(\omega_D t)$, where D is an arbitrary constant and $\omega_D \leq 0.003\ rad/s$, is attenuated on the output of 1000 times;
   - $\varphi_m \geq 50^\circ$ and $\omega_c \geq\ 0.1 rad/s$;
   - the control effort is limited.

Exercise 3

Consider the following closed-loop system



where $L(s) = \rho \dfrac{(s+2)(s-1)}{s(s^2+2s+2)}$.

1. Sketch the direct and inverse root loci.

2. Using the previous root loci, find the values of $\rho > 0$ for which the closed-loop system is asymptotically stable.

Exercise 4

Consider the following discrete time dynamical system

$$\begin{cases} x_1(k+1) = x_1(k) + u(k) \\ x_2(k+1) = -x_2(k) + u(k) \\ y(k) = x_1(k) + x_2(k) \end{cases}$$

1. Is the system stable, unstable or asymptotically stable?

2. Write the expression of the state and output trajectory generated by the initial state $x(0) = [x_1(0) \;\; x_2(0)]$ and by the input $u(k) = \text{sca}(k)$.

3. Sketch the zero-input state response to the initial states $x(0) = [1 \;\; 0]$ and $x(0) = [0 \;\; 1]$.

Exercise 5

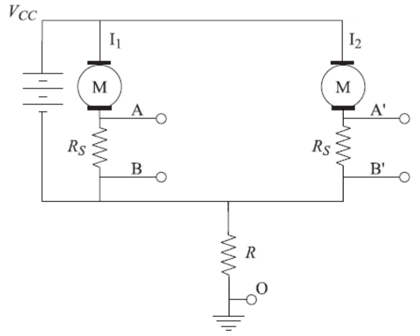
1. Consider an elastic servomechanism characterized by $J_m = 0.05\,Kgm^2$, $\rho = 2$, $K_{el} = 2000\,Nm/rad$, design a PI velocity controller.

2. Compute the sampling period for a digital realization of the PI velocity controller and determine the expression of its transfer function using Tustin method.

3. Sketch the diagram of a PI regulator including the anti-windup function.

Exercise 6

Consider the following circuit



where the currents flowing into the two motors can be measured through a differential measurement (v_{AB} or $v_{A'B'}$) or a single-ended measurement (v_{AO} or $v_{A'O}$). R is the resistance of the ground connection.

1. Compute the measurement errors related to a differential measurement and a single-ended measurement. What are the reasons of these errors?

2. Design an amplification circuit, characterised by an amplification factor of 10, for measurements v_{AB} and $v_{A'O}$, and show how it is connected to the system.

3. Modify the conditioning circuit of $v_{A'O}$ including a low-pass filter, with cut-off frequency $10\,Hz$, before the amplifier. Show how the circuit is connected to the system.