

Automatic Control A

(Prof. Bascetta)

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| Name:..... |
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Warnings:

- This file consists of **8** pages (including cover). All the pages should be signed.
- During the exam you are not allowed to exit the room for any other reason than handing your work or withdrawing from the exam.
- You are not allowed to withdraw from the exam during the first 30 minutes.
- During the exam you are not allowed to consult books or any kind of notes.
- You are not allowed to use calculators with graphic display.
- Solutions and answers can be given **either in English or in Italian**.
- Solutions and answers must be given **exclusively in the reserved space**. Only in the case of corrections, or if the space is not sufficient, use the back of the front cover.
- The clarity and the order of the answers will be considered in the evaluation.
- At the end of the test you have to **hand this file only**. Every other sheet you may hand will not be taken into consideration.

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Use this page ONLY in case of corrections or if the space reserved for some answers turned out to be insufficient

Exercise 1

Consider the following dynamical system

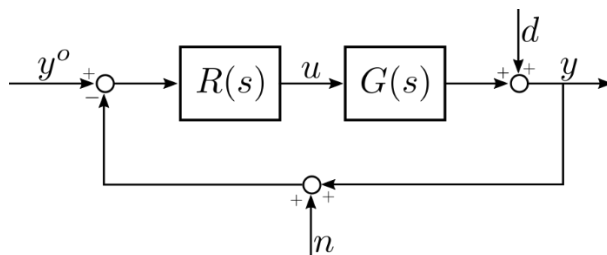
$$\begin{cases} \dot{x}_1 = -x_1 + e^{x_1} x_2 + u \\ \dot{x}_2 = -x_2 + x_3 \\ \dot{x}_3 = x_1 x_3 - x_3^2 - u \\ y = x_3 \end{cases}$$

1.1 Assuming $u(t) = \bar{u} = 0$, find the state and output equilibria.

1.2 Find, for each of the previous equilibria, the linearized system. Are these systems stable, unstable, or asymptotically stable? Are the corresponding equilibria stable, unstable or asymptotically stable?

Exercise 2

Consider the following control system



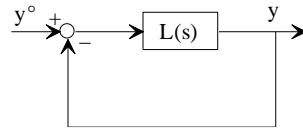
where $G(s) = 10 \frac{1 + 0.1s}{(1 + s)(1 + 0.02s)}$.

Compute the transfer function $R(s)$ of a controller, with order equal or less than 2, in such a way that:

- $|e_\infty| = 0$ for $y^\circ(t) = \text{sca}(t)$ and $d(t) = 0$ and $n(t) = 0$;
- $\varphi_m \geq 75^\circ$ and $\omega_c \geq 8 \text{ rad/s}$;
- a disturbance $d(t) = D \sin(\omega t)$, where D is an arbitrary constant and $\omega \leq 1 \text{ rad/s}$, is attenuated on y 10 times;
- a disturbance $n(t) = N \sin(\omega t)$, where N is an arbitrary constant and $\omega \geq 500 \text{ rad/s}$, is attenuated on y 100 times.

Exercise 3

Consider the following closed-loop system



where $L(s) = \rho \frac{s-1}{(s+1)^3}$.

3.1 Sketch the direct and inverse root loci.

3.2 Using the previous root loci, find the values of ρ for which the closed-loop system is asymptotically stable.

Exercise 4

Consider the following discrete time dynamical system

$$G(z) = \frac{z^2 - z + 2}{z^2}.$$

4.1 Is the discrete time system stable, unstable or asymptotically stable?

4.2 Find the analytical expression of the unit impulse response.

4.3 Compute the first five samples of the unit impulse response.

4.4 Show that the unit impulse response of a discrete time system with transfer function

$$G(z) = \frac{a_n z^n + a_{n-1} z^{n-1} + \dots + a_1 z + a_0}{z^n}$$

is zero for $k > n$ and that the values of the first $n+1$ samples are $a_n, a_{n-1}, \dots, a_1, a_0$.

Exercise 5

5.1 Write the expression of the transfer functions from motor torque to motor/load velocity of an elastic servomechanism described with the two-mass model. Explain the physical interpretation of the natural and locked frequency, and show how they are related to the parameters of the previous transfer functions.

5.2 Describe an experimental procedure that allows to identify the parameters of the previous transfer functions.

5.3 Explain the role of the a-dimensional parameter $\tilde{\omega}_{c_v}$ in the design of the velocity controller for an elastic servomechanism.

Exercise 6

6.1 How can we define a real-time system? How is the timeliness of a real-time system measured?

6.2 Explain the difference between a soft real-time system and a hard real-time system.

6.3 Describe the program, executed repeatedly every T_s milliseconds by a real-time system, required to implement a general digital regulator.