EXERCISE 1

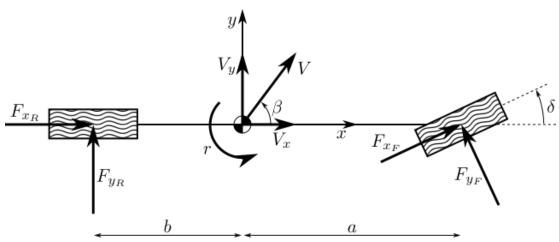
1. Consider a wheel rolling without slipping on the horizontal plane, keeping the sagittal plane in the vertical direction. Write the expression of the pure rolling constraint in the case of a fixed and a steerable wheel.

2. Are the previous constraints holonomic or nonholonomic? Motivate the answer using a mathematical proof.

3. Consider a single-track robot with front fixed wheel and rear steerable wheel. Assuming as configuration vector $\mathbf{q} = [x\ y\ \theta\ \phi]^T$, where $(x\ y\ \theta)$ is the robot pose and ϕ the steering angle, write the kinematic model of the robot without explicitly computing a base of Null $\left(A^T(\mathbf{q})\right)$.

EXERCISE 2

1. With reference to the following picture



write the Newton equations of the translational motion of the center of mass of the bicycle, and the Euler equation of the rotational motion, both referred to the body frame.

2. Consider now a bicycle with front and rear steering wheel. Write again the Newton equations of the translational motion of the center of mass of the bicycle, and the Euler equation of the rotational motion, both referred to the body frame.

3. During a curve the front wheel of a bicycle is subject to a longitudinal force $\bar{F}_{x_F} = 0.4\,\bar{F}_{z_F}$, and a lateral force $\bar{F}_{y_F} = 0.3\,\bar{F}_{z_F}$, where $\bar{F}_{z_F}$ is the vertical load on the wheel. Assume a unitary friction coefficient ($\mu = 1$).
   Keeping constant the longitudinal force and the vertical load, which is the maximum value achievable by the lateral force?
   Keeping constant the longitudinal force and doubling the vertical load, how does the maximum value change?

EXERCISE 3

1. Define a feasible path planning problem and an optimal path planning problem, stressing the main differences between the two problems.

2. Describe the algorithm to construct the simplified probabilistic roadmap used by sPRM.

3. How the previous algorithm should be modified in order to transform it into its optimal version?

EXERCISE 4

1. Consider a unicycle kinematic model. Assuming $z_1 = x$ and $z_2 = y$ as flat outputs, write the expression of the state and input variables as functions of the flat outputs and their derivatives.

2. How can the previous relations (flat representation of the unicycle kinematic model) be used to set up a trajectory tracking controller? Show a block diagram of the control system, including trajectory generation, controller and robot model.
   For each block specify the equations relating the inputs to the outputs. For trajectory generation consider a circle in the xy plane.

3. Illustrate the main pros and cons of the previous control solution with respect to a trajectory tracking controller based on feedback linearization.