Automatic Control - Project 2015/2016
Control of a Mobile Inverted Pendulum

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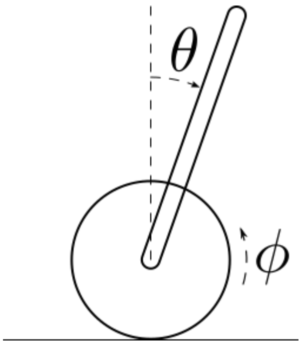


A Mobile Inverted Pendulum is a robotic platform whose kinematic and dynamic characteristics resemble a segway-like vehicle.
The equations of motion for the MIP are

$$\left(I_w + \left(m_w + m_r\right) R^2\right) \ddot{\phi} + \left(m_r R L \cos\theta\right) \ddot{\theta} = m_r R L \dot{\theta}^2 \sin\theta + \tau$$
$$\left(m_r R L \cos\theta\right) \ddot{\phi} + \left(I_r + m_r L^2\right) \ddot{\theta} = m_r g L \sin\theta - \tau$$

where ϕ and θ are the angular position of the wheel and the rod, respectively, τ is the motor torque, and

- m_w and m_r, are the mass of the wheel and of the road, respectively;
- I_w and I_r, are the inertia of the wheel and of the road, respectively;
- R, is the radius of the wheel;
- L, is the length from the end to the center of mass of the rod;
- g, is the gravity acceleration.

Assuming as state vector $\mathbf{x} = \begin{bmatrix}\phi & \theta & \dot{\phi} & \dot{\theta}\end{bmatrix}^T$ and as input $u = \tau$, the MIP can be described by the following linearised model

$$\begin{aligned}\dot{\mathbf{x}} &= \mathbf{A}\mathbf{x} + \mathbf{B}u \\ \mathbf{y} &= \mathbf{C}\mathbf{x}\end{aligned}$$

where

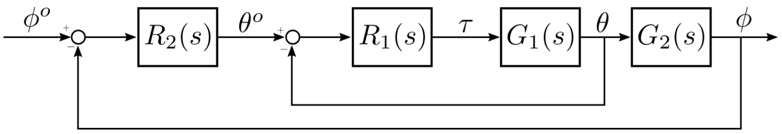
$$\mathbf{A} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & -\dfrac{m_r^2 R L^2 g}{R^2 m_w I_r + R^2 m_w m_r L^2 + R^2 m_r I_r + I_w I_r + I_w m_r L^2} & 0 & 0 \\ 0 & -\dfrac{-R^2 m_w m_r g L - R^2 m_r^2 g L - I_w m_r g L}{R^2 m_w I_r + R^2 m_w m_r L^2 + R^2 m_r I_r + I_w I_r + I_w m_r L^2} & 0 & 0 \end{bmatrix}$$

$$\mathbf{B} = \begin{bmatrix} 0 \\ 0 \\ \dfrac{I_r + m_r L^2 + m_r R L}{R^2 m_w I_r + R^2 m_w m_r L^2 + R^2 m_r I_r + I_w I_r + I_w m_r L^2} \\ -\dfrac{m_r R L + R^2 m_w + R^2 m_r + I_w}{R^2 m_w I_r + R^2 m_w m_r L^2 + R^2 m_r I_r + I_w I_r + I_w m_r L^2} \end{bmatrix}$$

$$\mathbf{C} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}$$

Consider the following points:

1. Using root locus, tune a controller, with transfer function $R_1(s) = \rho_R (s + z)/(s + p)$, that makes the closed-loop system asymptotically stable and characterised by two real poles at -2.
2. Using Bode criterion, design a controller $R_2(s)$ in such a way that the outer loop is characterised by a phase margin greater than 50°.



3. Compute a digital realization of the controllers designed in the previous steps, and simulate the digital control system assuming that the analogue to digital converter is characterised by 12 bit over a range of 360°, and the digital to analogue converter is characterised by 14 bit over a range of $\pm 5\,Nm$. Simulate the response of the system to a ramp with slope $0.1\,rad/s$.

4. Design a pole placement controller following these steps:
    - design a full-state feedback controller that places the poles of the closed-loop system at approximately -2. Simulate the response of the system to an initial condition characterised by $\theta = 0.1\,rad$ and all the other state variables equal to zero.
    - design a state observer that, using the positions of the wheel and the rod, estimates the states of the system. The dynamic of the state estimation error should be characterised by real poles five time faster than the poles of the closed-loop system. Compare the zero-input response of the system with the one obtained at the previous step.
    - introduce a reference input in the design of the pole placement controller to track, with zero steady-state error, a desired wheel position. Design the pole placement control law that places all the poles of the augmented system at approximately -2. Simulate the zero-state response of the system to a step of $0.5\,rad$.

Rules

1. Each student carries out the project individually, solving as many points as possible (but not necessarily all the points).
2. System parameters depend on student's data and can be found in the attached table.
3. When the project is over, each student sends by email a short pdf report describing the project (including student's data), all the Simulink models and the .m files required to initialize the models (Simulink models and Matlab files should be compatible with Matlab/Simulink R2015b), to luca.bascetta@polimi.it.
4. Project report and all Matlab/Simulink files should be sent at least three working days before the date of the exam at which the student is enrolled.

Identification number	Rod mass	Wheel mass	Rod moment of inertia	Wheel moment of inertia	L	R
842523	3,0582	19,4284	8,9946	0,0612	0,4000	0,2000
836679	3,0605	19,4434	9,0016	0,0612	0,4000	0,2000
835749	3,0609	19,4458	9,0027	0,0612	0,4000	0,2000
837484	3,7398	23,7586	10,9994	0,0748	0,4000	0,2000
841389	3,0586	19,4313	8,9960	0,0612	0,4000	0,2000
832716	3,7378	23,7463	10,9937	0,0748	0,4000	0,2000
859235	3,0514	19,3853	8,9747	0,0610	0,4000	0,2000
837260	3,7397	23,7580	10,9991	0,0748	0,4000	0,2000
823517	3,0659	19,4774	9,0173	0,0613	0,4000	0,2000
859390	3,7487	23,8151	11,0255	0,0750	0,4000	0,2000
842131	3,0583	19,4294	8,9951	0,0612	0,4000	0,2000
842398	3,7418	23,7713	11,0052	0,0748	0,4000	0,2000
837353	3,0603	19,4417	9,0008	0,0612	0,4000	0,2000
842390	3,7418	23,7713	11,0052	0,0748	0,4000	0,2000
853923	3,0535	19,3990	8,9810	0,0611	0,4000	0,2000
836850	3,7395	23,7570	10,9986	0,0748	0,4000	0,2000
836847	3,0605	19,4430	9,0014	0,0612	0,4000	0,2000
840681	3,0589	19,4331	8,9968	0,0612	0,4000	0,2000
841558	3,7414	23,7691	11,0042	0,0748	0,4000	0,2000
822544	3,7337	23,7201	10,9815	0,0747	0,4000	0,2000
837836	3,7399	23,7595	10,9998	0,0748	0,4000	0,2000
837046	3,7396	23,7575	10,9988	0,0748	0,4000	0,2000
837006	3,7396	23,7574	10,9988	0,0748	0,4000	0,2000
842411	3,0582	19,4287	8,9948	0,0612	0,4000	0,2000
833241	3,0619	19,4523	9,0057	0,0612	0,4000	0,2000
836646	3,7394	23,7565	10,9984	0,0748	0,4000	0,2000
823900	3,7343	23,7236	10,9832	0,0747	0,4000	0,2000
836473	3,0606	19,4440	9,0018	0,0612	0,4000	0,2000
836566	3,7394	23,7563	10,9983	0,0748	0,4000	0,2000
841306	3,7413	23,7685	11,0039	0,0748	0,4000	0,2000
836159	3,0608	19,4448	9,0022	0,0612	0,4000	0,2000
816644	3,7313	23,7049	10,9745	0,0746	0,4000	0,2000
854605	3,0533	19,3972	8,9802	0,0611	0,4000	0,2000
836219	3,0607	19,4446	9,0021	0,0612	0,4000	0,2000
841834	3,7415	23,7698	11,0046	0,0748	0,4000	0,2000
842740	3,7419	23,7722	11,0056	0,0748	0,4000	0,2000
836169	3,0607	19,4448	9,0022	0,0612	0,4000	0,2000
841725	3,0585	19,4304	8,9956	0,0612	0,4000	0,2000
836406	3,7393	23,7558	10,9981	0,0748	0,4000	0,2000
837426	3,7398	23,7585	10,9993	0,0748	0,4000	0,2000
836199	3,0607	19,4447	9,0022	0,0612	0,4000	0,2000
842535	3,0582	19,4284	8,9946	0,0612	0,4000	0,2000
842017	3,0584	19,4297	8,9952	0,0612	0,4000	0,2000
836921	3,0604	19,4428	9,0013	0,0612	0,4000	0,2000
837123	3,0604	19,4423	9,0011	0,0612	0,4000	0,2000
837790	3,7399	23,7594	10,9997	0,0748	0,4000	0,2000
835221	3,0611	19,4472	9,0033	0,0612	0,4000	0,2000
841979	3,0584	19,4298	8,9953	0,0612	0,4000	0,2000

823756	3,7342	23,7232	10,9830	0,0747	0,4000	0,2000
841608	3,7415	23,7693	11,0043	0,0748	0,4000	0,2000
836223	3,0607	19,4446	9,0021	0,0612	0,4000	0,2000
836840	3,7395	23,7570	10,9986	0,0748	0,4000	0,2000
838498	3,7402	23,7612	11,0006	0,0748	0,4000	0,2000
836194	3,7393	23,7553	10,9978	0,0748	0,4000	0,2000
837295	3,0603	19,4419	9,0009	0,0612	0,4000	0,2000
837874	3,7399	23,7596	10,9998	0,0748	0,4000	0,2000
841461	3,0586	19,4311	8,9959	0,0612	0,4000	0,2000
835641	3,0610	19,4461	9,0028	0,0612	0,4000	0,2000
842310	3,7417	23,7711	11,0051	0,0748	0,4000	0,2000
837727	3,0601	19,4407	9,0003	0,0612	0,4000	0,2000
852440	3,7459	23,7972	11,0172	0,0749	0,4000	0,2000
837795	3,0601	19,4406	9,0003	0,0612	0,4000	0,2000
838712	3,7403	23,7618	11,0008	0,0748	0,4000	0,2000
838568	3,7402	23,7614	11,0007	0,0748	0,4000	0,2000
842435	3,0582	19,4286	8,9947	0,0612	0,4000	0,2000
836698	3,7395	23,7566	10,9984	0,0748	0,4000	0,2000
858283	3,0518	19,3878	8,9758	0,0610	0,4000	0,2000