

Control of Industrial and Mobile Robots

PROF. ROCCO, BASCETTA

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NAME:

UNIVERSITY ID NUMBER:

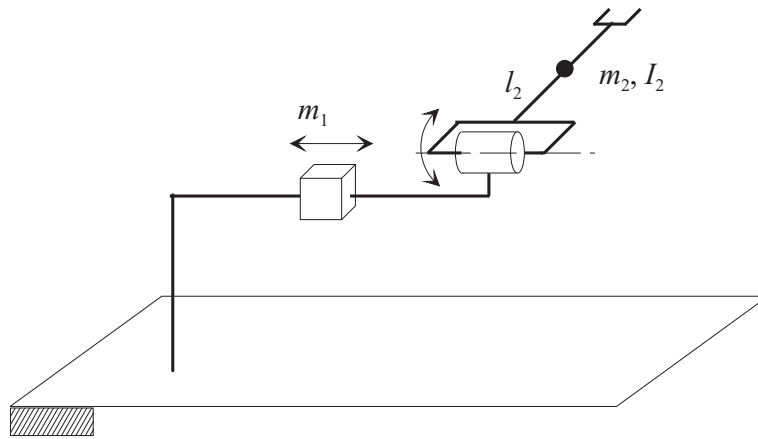
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Warnings

- This file consists of **10** pages (including cover).
- During the exam you are not allowed to exit the room for any other reason than handing your work or withdrawing from the exam.
- You are not allowed to withdraw from the exam during the first 30 minutes.
- During the exam you are not allowed to consult books or any kind of notes.
- You are not allowed to use calculators with graphic display.
- Solutions and answers can be given **either in English or in Italian**.
- Solutions and answers must be given **exclusively in the reserved space**. Only in the case of corrections, or if the space is not sufficient, use the back of the front cover.
- The clarity and the order of the answers will be considered in the evaluation.
- At the end of the test you have to **hand this file only**. Every other sheet you may hand will not be taken into consideration.

EXERCISE 1

1. Consider the manipulator sketched in the picture:



Find the expression of the inertia matrix $\mathbf{B}(\mathbf{q})$ of the manipulator.

EXERCISE 2

1. Explain what is meant with kinematic scaling of a trajectory and write the general expression of a trajectory in the form (parameterized) which is used in such scaling.

2. The parametric form of a harmonic trajectory for kinematic scaling is given by:

$$\sigma(\tau) = \frac{1}{2} (1 - \cos(\pi\tau))$$

Find the expressions of the maximum velocity and maximum acceleration for such trajectory in terms of the positioning time T and the total displacement h .

3. Consider the design of a harmonic trajectory from $q_i = 10$ to $q_f = 30$, with $\dot{q}_{\max} = 10$ and $\ddot{q}_{\max} = 20$. Find the minimum positioning time.

4. For the harmonic trajectory computed in this exercise, sketch the plot of the speed $\dot{q}(t)$, assuming that the trajectory starts at time $t_i = 0$.

EXERCISE 3

1. Given a kinematic constraint in Pfaffian form

$$X(\mathbf{q})\dot{x} + Y(\mathbf{q})\dot{y} + Z(\mathbf{q})\dot{z} = 0$$

where $\mathbf{q} = [x \ y \ z]$ is the configuration vector. Illustrate the necessary and sufficient condition for this constraint to be holonomic.

2. Consider the following kinematic constraints

$$\dot{q}_1 + q_1\dot{q}_2 - \dot{q}_3 = 0 \quad q_1\dot{q}_1 - q_2^2\dot{q}_3 = 0$$

where $\mathbf{q} \in \mathbb{R}^3$ is the configuration vector. Determine, using the necessary and sufficient condition, if each of these constraints by itself is holonomic or nonholonomic.

3. Consider two mobile robots, whose configurations are described by $\mathbf{q} \in \mathbb{R}^3$, and whose motion is described by $\dot{q}_1 + q_1 \dot{q}_2 - \dot{q}_3 = 0$ for the first robot, and $q_1 \dot{q}_1 - q_2^2 \dot{q}_3 = 0$ for the second robot. Does the following kinematic model

$$\dot{\mathbf{q}} = \begin{bmatrix} -q_1 \\ 1 \\ 0 \end{bmatrix} u_1 + \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} u_2$$

describe the motion of the first robot, and the following kinematic model

$$\dot{\mathbf{q}} = \begin{bmatrix} q_2^2 \\ 0 \\ q_1 \end{bmatrix} u_1 + \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} u_2$$

the motion of the second robot?
Clearly motivate the answer.

4. Considering the second constraint ($a^T(\mathbf{q})\dot{\mathbf{q}} = q_1 \dot{q}_1 - q_2^2 \dot{q}_3 = 0$), two vectors in the null of $a^T(\mathbf{q})$ are

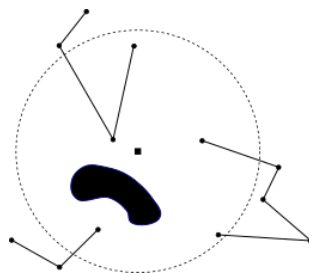
$$\mathbf{g}_1 = \begin{bmatrix} q_2^2 \\ q_1 \\ 0 \\ 1 \end{bmatrix} \quad \mathbf{g}_2 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$

Compute the vector field representing the motion that is locally constrained by $q_1 \dot{q}_1 - q_2^2 \dot{q}_3 = 0$.

EXERCISE 4

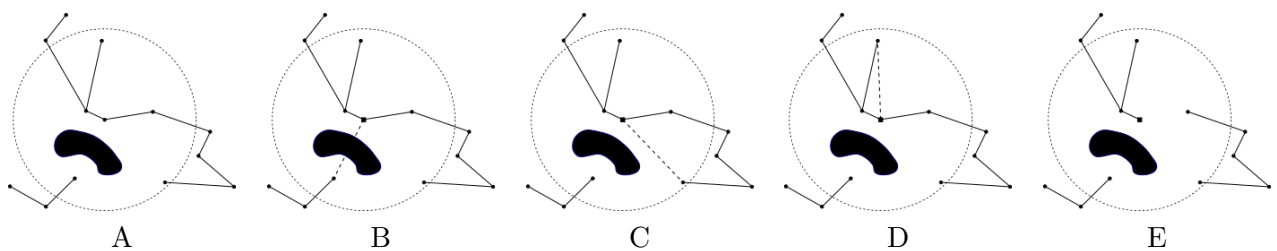
- Write and explain the pseudocode of the algorithm to construct the probabilistic roadmap used by PRM.

- Consider the i -th iteration of PRM as depicted in the figure below



where the black square is the current \mathbf{q}_{rand} , the black blob region is an obstacle, the black dashed circle centred in \mathbf{q}_{rand} is the region defining the Near set, and black dots and segments are nodes and edges in the actual V and E sets, respectively.

Put the following pictures in the correct order, according to the execution of the algorithm across the nodes belonging to the Near set (black dashed edges represent connection attempts that are discarded).



3. How could the path resulting from PRM be used to speed up the computation of RRT*? Clearly motivate the answer.

4. Consider again an RRT* planner, what are the characteristics a cost function should satisfy in order to be included into the planner?