Control of Mobile Robots

PROF. BASCETTA

JANUARY 24, 2022

NAME:
UNIVERSITY ID NUMBER:

SIGNATURE: ______________________________

Warnings

- This file consists of **8** pages (including cover).
- During the exam you are not allowed to exit the room for any other reason than handing your work or withdrawing from the exam.
- You are not allowed to withdraw from the exam during the first 30 minutes.
- During the exam you are not allowed to consult books or any kind of notes.
- You are not allowed to use calculators with graphic display.
- Solutions and answers can be given **either in English or in Italian**.
- Solutions and answers must be given **exclusively in the reserved space**. Only in the case of corrections, or if the space is not sufficient, use the back of the front cover.
- The clarity and the order of the answers will be considered in the evaluation.
- At the end of the test you have to **hand this file only**. Every other sheet you may hand will not be taken into consideration.

EXERCISE 1

1. Given the kinematic constraint

$$\dot{q}_1 - q_1\dot{q}_2 + 4\dot{q}_3 = 0$$

where $\mathbf{q} = \begin{bmatrix} q_1 & q_2 & q_3 \end{bmatrix}$ is the configuration vector. Determine, using the necessary and sufficient condition, if this constraint is holonomic or nonholonomic.

2. Given the kinematic constraint

$$2\dot{q}_2 - q_1\dot{q}_3 = 0$$

where $\mathbf{q} = \begin{bmatrix} q_1 & q_2 & q_3 \end{bmatrix}$ is the configuration vector. Determine, using the necessary and sufficient condition, if this constraint is holonomic or nonholonomic.

3. Is the system of two constraints

$$\dot{q}_1 - q_1\dot{q}_2 + 4\dot{q}_3 = 0 \qquad 2\dot{q}_2 - q_1\dot{q}_3 = 0$$

holonomic or nonholonomic? Motivate the answer analysing the accessibility distribution.

EXERCISE 2

Consider a unicycle robot whose mass and yaw inertia vary with time.

1. Write the expression of the Lagrangian function, of matrix $S(\mathbf{q})$, and $A(\mathbf{q})$.

2. Write the dynamic model using Lagrange equations.

3. How does the time-varying mass and inertia affect the computation of the tyre-ground interaction model? Consider a linear interaction model.

EXERCISE 3

1. Write and explain the pseudocode of the algorithm to construct the probabilistic roadmap used by PRM.

2. Explain how the previous algorithm has to be modified in order to obtain sPRM and PRM* algorithms.

3. In PRM the Near function is used to determine the nodes that belong to the Nearest neighbour. Give a mathematical definition of the Near function used by PRM, explaining how the function works. Show two other ways of computing the Nearest neighbour.

EXERCISE 4

Consider a simplified version of the rear-wheel drive bicycle model

$$\dot{x} = v\cos\theta$$
$$\dot{y} = v\sin\theta$$
$$\dot{\theta} = \frac{v}{\ell}\tan\phi$$

where (x, y, θ) is the position and orientation of the vehicle, v the linear velocity, and ϕ the steering angle.

1. Write the expression of the feedback linearising law for this model.

2. Is the previous linearising feedback affected by any singularity? Does it introduce any hidden dynamics? If yes, which are the states that belong to the hidden dynamics? Clearly motivate the answer.

3. Write the equations of the dynamical system representing the closed-loop system obtained connecting the model with the controller.