EXERCISE 1

1. Explain how kinematic constraints can be classified, listing all the classes and describing the main characteristics of each class.

2. Explain the difference between holonomic and nonholonomic constraints (in particular their effects on the motion of the system), and illustrate an example of a physical system characterised by holonomic constraints and of a physical system characterised by nonholonomic constraints.

3. Consider a mobile robot whose configuration is described by vector $\mathbf{q} = [q_1,\ q_2,\ q_3,\ q_4]$, and characterized by the following two constraints

$$\dot{q}_1 + q_1\dot{q}_2 + \dot{q}_3 = 0 \qquad \dot{q}_2 + q_2\dot{q}_3 = 0$$

Does the following kinematic model

$$\dot{\mathbf{q}} = \begin{bmatrix} q_1 q_2 - 1 \\ -q_2 \\ 1 \\ 0 \end{bmatrix} u_1 + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} u_2$$

describe the motion of this mechanical system? Clearly motivate the answer.

EXERCISE 2

Consider a rear-wheel drive bicycle robot under the following assumptions:

- braking force can be neglected;
- sideslip and steering angles are small (i.e., $\cos x \approx 1$ and $\sin x \approx x$);
- force-slip relation is linear;
- the absolute velocity is slowly varying.

1. Write the configuration vector and the equations describing the kinematic model. Explain the meaning of each symbol used in the equations.

2. Write the equations describing the dynamic model considering the previous assumptions and using as state variables sideslip and yaw rate. Explain the meaning of each symbol used in the equations.

3. Consider now a force-slip relation for the lateral force that is linear up to the maximum available force (given by friction), and then saturates to the friction force. Write the expression of the front/rear lateral forces and the equations describing the dynamic model. Explain the meaning of each symbol used in the equations.

ESERCIZIO 3

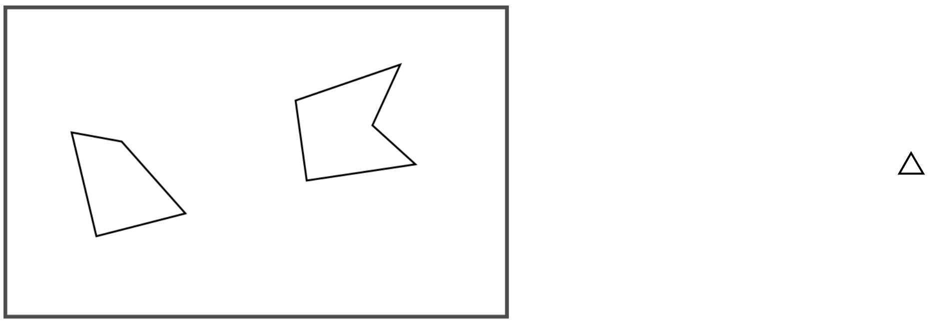
1. A navigation system includes a planner (or global planner) and a controller (or local planner). Answer to the following questions (including motivations, and examples illustrating the answer):

    (a) what are the main characteristics and aims of the planner and of the controller?

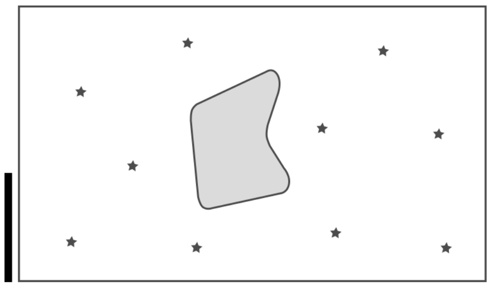
    (b) can we navigate using only a planner?

    (c) can we navigate using only a controller?

2. Consider the map shown in the figure, composed of a rectangular space and two obstacles, and a robot with the shape of an equilateral triangle. Draw on the figure $\mathcal{Q}_{free}$ and $\mathcal{Q}_{obs}$ (do not care about the exact size of the robot).



3. Using sPRM and considering the map shown in the figure, where stars represent vertex and the thick segment on the left side represents the length of the radius of the ball defining the set of near nodes, determine the roadmap and draw it on the figure.



ESERCIZIO 4

1. Consider the problem of deriving a trajectory tracking controller for a unicycle robot, illustrate the main steps required to determine the system that describes the error dynamics (i.e., $\dot{\mathbf{e}} = \dots$).

2. Illustrate the main steps required to determine a linear controller based on the linearized error dynamics determined at the previous step.

3. Considering the control law determined at the previous step, under which conditions the origin of the nonlinear error dynamics is asymptotically stable? Motivate the answer.