EXERCISE 1

1. Consider a wheel rolling without slipping on the horizontal plane, keeping the sagittal plane in the vertical direction. Write the expression of the pure rolling constraint in the case of a fixed and a steerable wheel.

2. Are the previous constraints holonomic or nonholonomic? Motivate the answer with a mathematical proof.

3. Consider a single-track robot with front fixed wheel and rear steerable wheel. Assuming as configuration vector $\mathbf{q} = [x\ y\ \theta\ \phi]^T$, where $(x\ y\ \theta)$ is the robot pose and ϕ the steering angle, write the kinematic model of the robot without explicitly computing a base of Null $\left(A^T(\mathbf{q})\right)$.

EXERCISE 2

1. What are the approaches that can be used to model the wheel-ground interaction? List them and explain the differences between them.

2. The longitudinal force in the brush or Fiala model is given by the following expression

$$F_x = \begin{cases} C_x \sigma_x \left(-1 + \frac{|\sigma_x|}{\sigma_{x_{sl}}} - \frac{\sigma_x^2}{3\sigma_{x_{sl}}^2} \right) & |\sigma_x| < \sigma_{x_{sl}} \\ -\mu F_z \text{sign}\left(\sigma_x\right) & |\sigma_x| \geq \sigma_{x_{sl}} \end{cases}$$

Explain the meaning of each symbol and of the wheel-ground interaction model equation.

3. Write the friction circle constraint and explain its meaning.

ESERCIZIO 3

1. Define a feasible path planning problem and an optimal path planning problem, stressing the main differences between the two problems.

2. Describe the algorithm to construct the <u>simplified</u> probabilistic roadmap used by sPRM.

3. How the previous algorithm should be modified in order to transform it into its optimal version?

ESERCIZIO 4

1. Consider a robot described by the unicycle kinematic model, and a point P related to the unicycle wheel contact point (x, y) by the following relations

$$x_P = x + \varepsilon \cos\theta \qquad y_P = y + \varepsilon \sin\theta$$

   Show how a feedback control law that linearises the unicycle model can be derived.

2. Design a trajectory tracking controller based on the linearising law introduced in the previous step, and draw a block diagram of the entire control system, explaining how it can be tuned.

3. Explain why the control system designed in the previous step cannot control the robot heading.