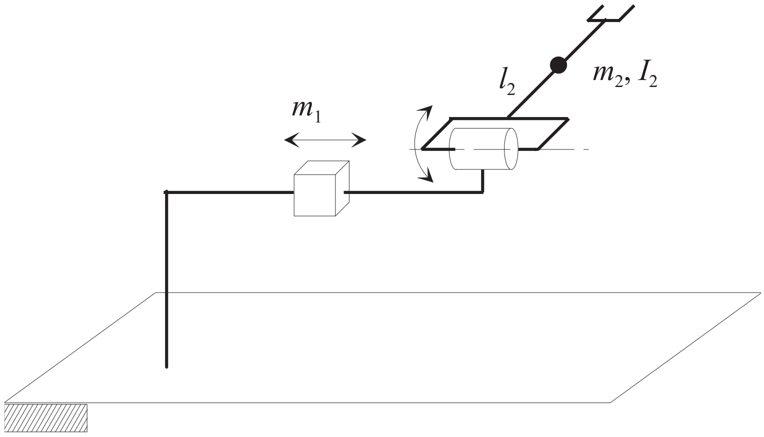
EXERCISE 1

1. Consider the manipulator sketched in the picture:



Find the expression of the inertia matrix $\mathbf{B}(\mathbf{q})$ of the manipulator.

2. Write the complete dynamic model for this manipulator.

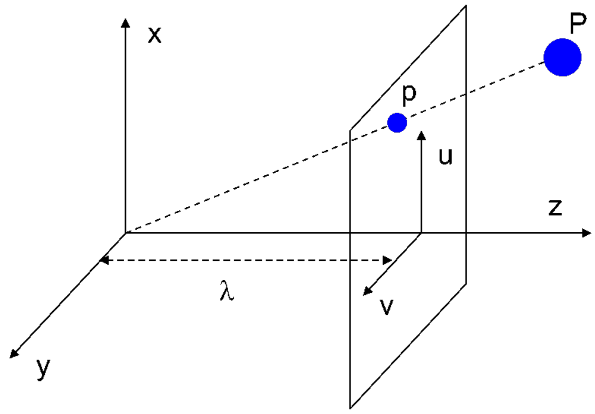
3. Consider the adoption of an inverse dynamics controller for this manipulator. Write the expressions of the two control variables.

4. Assume that the inverse dynamics controller assigns the same dynamics in closed loop to both joints of the manipulator. Compute the gains of the controller in such a way that both eigenvalues are equal to -10.

EXERCISE 2

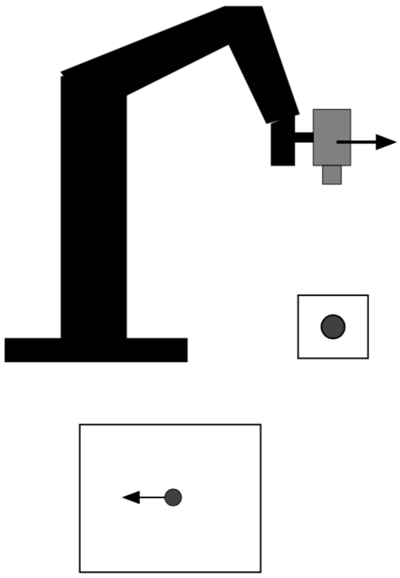
Consider the control of a manipulator with vision sensors.

1. Explain what is the "perspective projection" method and, making reference to the following picture, write the related formulas.



2. Making reference to the following picture, explain what is the interaction matrix in the context of visual control, specifying precisely:

   - the variables that are related by the interaction matrix
   - the size of the interaction matrix
   - the variables upon which the interaction matrix depends



3. Explain what is the image Jacobian and what is its relation with the interaction matrix.

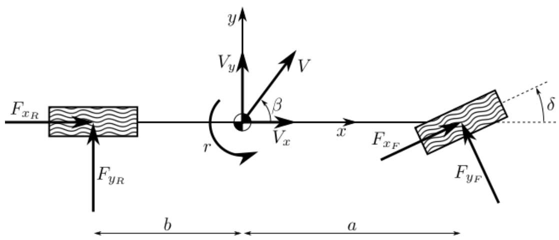
4. Sketch the block diagram of an image-based look-and-move control scheme, specifying the control law in the image space in terms of the image Jacobian.

EXERCISE 3

1. Write the expression of the two constraints that characterize a bicycle robot in Pfaffian form, and compute a basis of Null $\left(A^T\left(\mathbf{q}\right)\right)$.

2. Show that (using a mathematical proof) the two constraints considered in step 1 are nonholonomic constraints.

3. With reference to the following picture



write the Newton equations of the translational motion of the center of mass of the bicycle, and the Euler equation of the rotational motion, both referred to the body frame.

4. During a curve the front wheel of a bicycle is subject to a longitudinal force $\bar{F}_{xF} = 0.4\,\bar{F}_{zF}$, and a lateral force $\bar{F}_{yF} = 0.3\,\bar{F}_{zF}$, where $\bar{F}_{zF}$ is the normal load on the wheel.
Keeping constant the longitudinal force and the normal load, which is the maximum value achievable by the lateral force?
Keeping constant the longitudinal force and doubling the normal load, how does the maximum value change?

EXERCISE 4

1. Consider kinodynamic RRT* planner applied to the kinematic model of a unicycle robot with the aim of minimizing the duration of the trajectory while penalizing the control effort.
   Write the problem that must be solved to compute an edge of the tree.

2. With reference to RRT*, explain the rewire procedure.
   Using an example, show how the rewire procedure modifies the tree.

3. Consider the kinematic model of a unicycle robot, and a point P at a distance p from the wheel contact point along the direction of the velocity vector. Write the expression of the feedback linearizing controller and draw the block diagram of the system composed by the robot model and the controller.

4. An experiment is executed on the real robot, performing a step response first on v_{xP} (with $v_{yP} = 0$), and then on v_{yP} (with $v_{xP} = 0$). Due to unmodelled dynamics, the two step responses appears as the response of a first order system (instead of an integrator), with unitary gain and a settling time of 0.05 seconds.
Design and tune a trajectory tracking controller. Motivate how you select the crossover frequency.