Automatic Control
Final exam example
Prof. Luca Bascetta

Exercise 1

Consider a continuous time linear and time invariant dynamic system

$$\dot{\mathbf{x}}(t) = \mathbf{A}\mathbf{x}(t) + \mathbf{B}u(t)$$
$$\mathbf{y}(t) = \mathbf{C}\mathbf{x}(t)$$

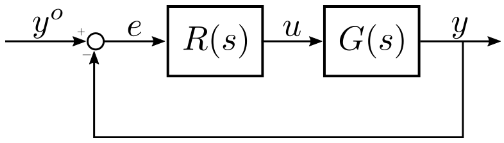
1. Which of the state space matrices $\mathbf{A}$, $\mathbf{B}$, $\mathbf{C}$ affect the stability of the system? Why?
2. Which is the definition of controllability?
3. Find the values of parameter $\alpha \in \mathbb{R}$ for which the following system is asymptotically stable

$$\mathbf{A} = \begin{bmatrix} -\alpha & 1 & 0 \\ -2 & 0 & -1 \\ 1 & 0 & 0 \end{bmatrix} \qquad \mathbf{B} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$
$$\mathbf{C} = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix}$$

4. Find the values of parameter $\alpha \in \mathbb{R}$ for which the previous system is completely controllable.

Exercise 2

Consider the following closed loop system



where $G(s) = \dfrac{1}{(1+s)(1+0.1s)^2}$.

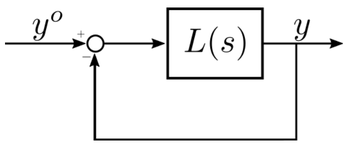
1. Compute the transfer function of the controller $R(s)$ in such a way that:
   - $|e_\infty| \leq 0.12$ for $y^o(t) = \text{sca}(t)$
   - the phase margin is greater or equal to $60°$
   - the crossover frequency is greater or equal to $3\,rad/s$.
2. Using the controller previously designed, draw the qualitative plot of the response of the closed loop system to a unit step on the reference signal.
3. Assuming that

$$G(s) = \frac{1}{(1+s)(1+0.1s)^2}e^{-\tau s} \quad \tau > 0$$

   and using the controller previously designed, compute the maximum value of τ for which the closed loop system is asymptotically stable.

Exercise 3

Consider the following closed loop system



where $L(s) = \rho\dfrac{s-1}{(s+1)^2(s+2)}$.

1. Sketch the direct root locus.
2. Sketch the inverse root locus.
3. Using the previous root loci, find the values of ρ for which the closed loop system is asymptotically stable.
4. Select a value $\rho > 0$ for which the closed loop system has a pair of real and coincident poles. Is the closed loop system asymptotically stable for this value?

Exercise 4

Consider the following discrete time dynamic system

$$G(z) = \frac{2z+1}{z^2 - z - 6}$$

1. Compute the gain and type of the transfer function.
2. Is the discrete time system stable, unstable or asymptotically stable?
3. Compute the analytic expression ($y(k) = \dots$) of the unit step response.
4. Compute the first 5 samples of the unit ramp response.

Exercise 5

1. Write the expression of a general trajectory $q(t)$ in normalised form suitable for kinematic scaling.
2. Write the expression of the derivatives of $q(t)$.
3. Given the normalized form of an harmonic trajectory

   $$\sigma(\tau) = \frac{1}{2}\left(1 - \cos(\pi\tau)\right)$$

   compute the maximum velocity and maximum acceleration for the general harmonic trajectory.
4. Consider an harmonic trajectory characterised by $q_i = 0$, $q_f = 4$, $\dot{q}_i = \dot{q}_f = 0$, $\dot{q}_{max} = 30$, $\ddot{q}_{max} = 50$. Compute the minimum trajectory duration.

Exercise 6

Consider the P/PI control of an elastic servomechanism characterised by $J_m = 0.05\,Kg\,m^2$, $\rho = 3$, $K_{el} = 1500\,Nm/rad$.

1. Assuming that the rotor is locked, compute the frequency at which the load oscillates, and compare it with the frequency at which the whole system (motor, transmission and load) oscillates when the rotor is not locked.
2. Design a PI velocity controller for the servomechanism.
3. Find a suitable sampling time for the digital implementation of the previous regulator.
4. Design a circuit for a first order passive anti-aliasing filter, assuming that the signal is single-ended. Compute the phase margin decrement due to the filter.
5. Assume that an operator starts the control system pushing a 'start' button, and the control system is up and running until the operator pushes the 'stop' button or a failure signal is received. Write the ladder diagram that allows to implement this behaviour on a PLC.
6. Describe the main steps executed by a PLC at every cycle.