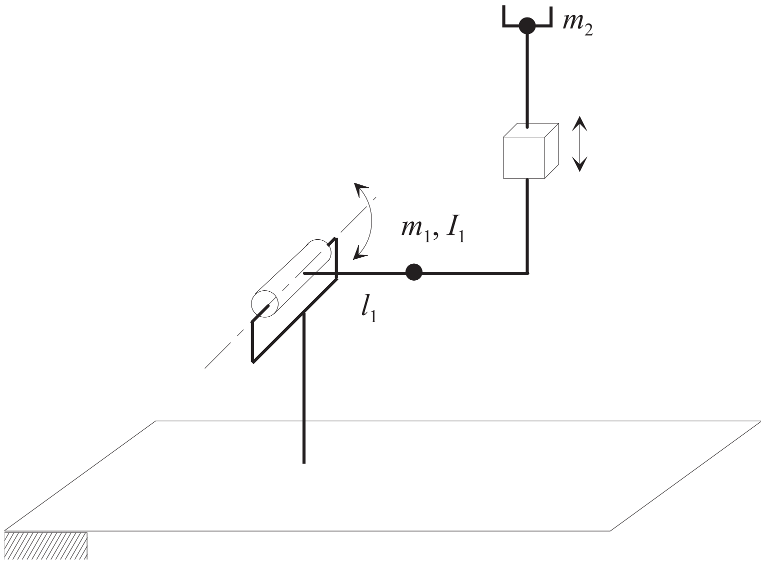
EXERCISE 1

1. Consider the manipulator sketched in the picture, where the mass of the second link is assumed to be concentrated at the end-effector:



Find the expression of the inertia matrix $\mathbf{B}(\mathbf{q})$ of the manipulator.

2. Compute the matrix $\mathbf{C}(\mathbf{q}, \dot{\mathbf{q}})$ of the Coriolis and centrifugal terms[1] for this manipulator.

3. Check that the expression $\dot{\mathbf{q}}^T \mathbf{N}(\mathbf{q}, \dot{\mathbf{q}}) \dot{\mathbf{q}} = 0$, where $\mathbf{N}(\mathbf{q}, \dot{\mathbf{q}}) = \dot{\mathbf{B}}(\mathbf{q}) - 2\mathbf{C}(\mathbf{q}, \dot{\mathbf{q}})$, is true in this case.

4. Write the complete expression of the kinetic energy for this specific manipulator.

[1]The general expression of the Christoffel symbols is $c_{ijk} = \frac{1}{2}\left(\frac{\partial b_{ij}}{\partial q_k} + \frac{\partial b_{ik}}{\partial q_j} - \frac{\partial b_{jk}}{\partial q_i}\right)$

EXERCISE 2

1. Consider a kinematically redundant manipulator. Explain what the “null-space motions” are, and write an expression for the null-space motions, explaining the meaning of each symbol used.

2. Define what is a repeatable, or cyclic, method for kinematic redundancy resolution. Explain what are the advantages in using a repeatable method.

3. Write the expression of the extended Jacobian method for redundancy resolution and explain why it is repeatable.

4. Consider now motion planning of the end-effector position. Select as an initial point $\mathbf{p}_i = [0, 0, 1]$ and as a final point $\mathbf{p}_f = [2, 2, 2]$. Write the expression of a segment connecting the initial and the final points, parameterized with the natural coordinate.

   If the end-effector task is expressed in terms of position only, what is the minimum number of joints for the manipulator to be redundant with respect to this task?

EXERCISE 3

Consider a rear-wheel drive bicycle robot under the following assumptions:

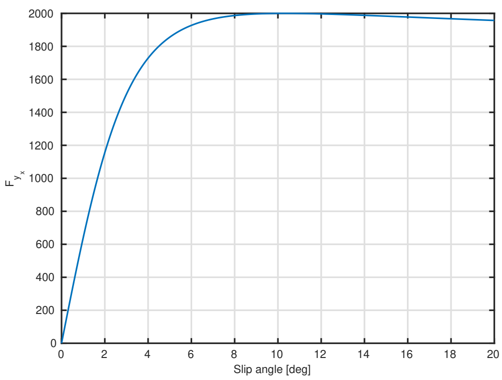
- braking force can be neglected;
- sideslip and steering angles are small (i.e., $\cos x \approx 1$ and $\sin x \approx x$);
- force-slip relation is linear;
- the absolute velocity is slowly varying.

1. Write the configuration vector and the equations describing the kinematic model. Explain the meaning of each symbol used in the equations.

2. Write the equations describing the dynamic model considering the previous assumptions and using as state variables sideslip and yaw rate. Explain the meaning of each symbol used in the equations.

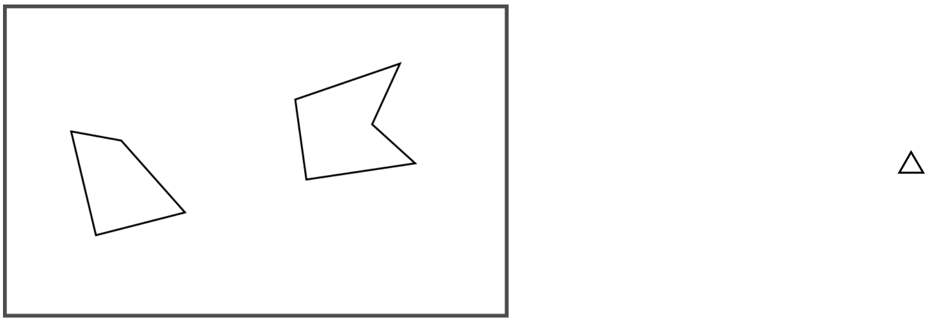
3. Consider now a force-slip relation for the lateral force that is linear up to the maximum available force (given by friction), and then saturates to the friction force. Write the expression of the front/rear lateral forces and the equations describing the dynamic model. Explain the meaning of each symbol used in the equations.

4. Determine the parameters of the force-slip relation introduced in the previous step according to the figure below, assuming $F_z = 2000\,\mathrm{N}$ and $\mu = 1$.

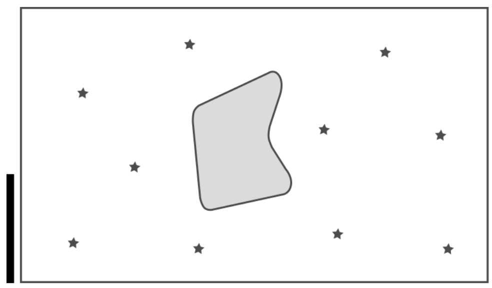


EXERCISE 4

1. Consider the map shown in the figure, composed of a rectangular space and two obstacles, and a robot with the shape of an equilateral triangle. Draw on the figure $\mathcal{Q}_{free}$ and $\mathcal{Q}_{obs}$ (do not care about the exact size of the robot).



2. Using sPRM and considering the map shown in the figure, where stars represent vertex and the thick segment on the left side represents the length of the radius of the ball defining the set of near nodes, determine the roadmap and draw it on the figure.



3. Consider the problem of deriving a trajectory tracking controller for a unicycle robot, illustrate the main steps required to determine the system that describes the error dynamics (i.e., $\dot{\mathbf{e}} = \dots$).

4. Illustrate the main steps required to determine a linear controller based on the linearized error dynamics determined at the previous step.