EXERCISE 1 (Kinematics)

1. Considering a disk rolling without slipping on the horizontal plane, keeping the sagittal plane in the vertical direction. Write the expression of the pure rolling constraint explaining its physical meaning.

2. Show that the pure rolling constraint is a nonholonomic constraint.

3. Is the constraint $\dot{x} + (1 - x)\,\dot{y} + \dot{z} = 0$ holonomic or nonholonomic? Motivate the answer.

EXERCISE 2 (Dynamics)

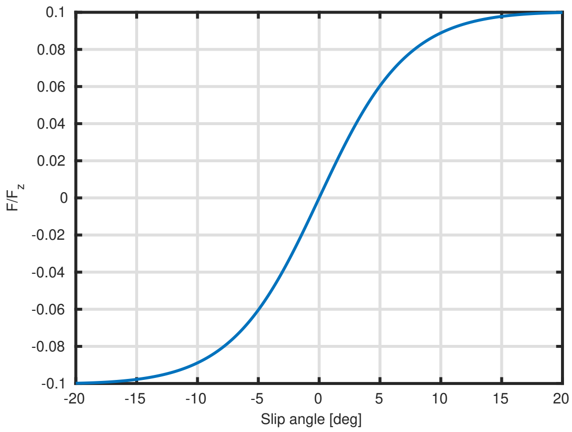
1. Draw the diagram of a bicycle vehicle, showing all the forces acting on it. Write the analytic expression of the force and moment balances.

2. Explain the physical meaning of the front/rear slip angles. How are these angles related to the vehicle dynamics variables (speed, steer, yaw rate, ...)?

3. Assume that the front/rear lateral forces are described by the following linear relations

$$F_{y_F} = -C_{\alpha_F}\alpha_F \qquad F_{y_R} = -C_{\alpha_R}\alpha_R$$

while the experimental lateral force-slip relation is given (for both F_{y_F} and F_{y_R}) by the curve in the following figure



Compute the values of C_{α_F} and C_{α_R}, and explain their relation with the experimental curve.

ESERCIZIO 3 (Planning)

1. Consider a unicycle kinematic model, assuming as flat outputs variables $z_1 = x$ and $z_2 = y$, write the expressions of the flatness transformations.

2. Given as initial state $\mathbf{q}(0) = [x_i, y_i, \theta_i]$ and final state $\mathbf{q}(t_f) = [x_f, y_f, \theta_f]$, write the analytical expression of the trajectory planned using the flatness transformation and explain how this trajectory can be computed.

3. Explain how the previous procedure can be modified in order to introduce a cost function and plan a trajectory that minimizes this cost function as well.

ESERCIZIO 4 (Control)

1. The following control law is used to linearise the dynamics of a unicycle kinematic model

$$v = v_{xP} \cos\theta + v_{yP} \sin\theta$$
$$\omega = \frac{v_{yP} \cos\theta - v_{xP} \sin\theta}{\varepsilon}$$

where v_{xP} and v_{yP} are the velocities of a point P at a distance ε from the wheel contact point along the sagittal axis of the vehicle. Compute the equations of the closed loop system resulting from the application of the previous law.

2. Design a trajectory tracking controller for a unicycle robot based on the linearising law introduced in Step 1. Draw a block diagram of the entire control system and explain how it can be tuned.

3. Explain why the control system designed in Step 2 cannot be used to regulate the pose of the robot.