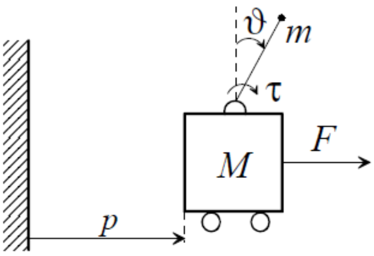
Automatic Control - Laboratory 2
Pole placement
Prof. Luca Bascetta



Consider a cart with an inverted pendulum.
The cart, having mass equal to M, is in rectilinear motion due to a force F. The inverted pendulum is a beam, having length l and whose mass can be neglected, moved by a torque τ. At the end of the beam there is a lumped mass m.
Applying Lagrange equations, one can derive the motion equations of the cart-pendulum system

$$(M+m)\ddot{p} - ml\dot{\vartheta}^2 \sin(\vartheta) + ml\ddot{\vartheta}\cos(\vartheta) = F$$
$$ml^2\ddot{\vartheta} + ml\ddot{p}\cos(\vartheta) - mgl\sin(\vartheta) = \tau$$

where p and ϑ are the position of the cart and the of the beam, respectively. Linearising the model around an equilibrium characterised by zero linear/angular velocity and position, and by zero force and torque, one obtains

$$\delta\ddot{p} = -\frac{m}{M}g\delta\vartheta + \frac{1}{M}\delta F - \frac{1}{lM}\delta\tau$$
$$\delta\ddot{\vartheta} = \frac{g}{l}\frac{M+m}{M}\delta\vartheta - \frac{1}{lM}\delta F + \frac{1}{l^2}\frac{M+m}{Mm}\delta\tau$$

where $M = 10$, $m = 1$, $l = 1$, $g = 9.8$.

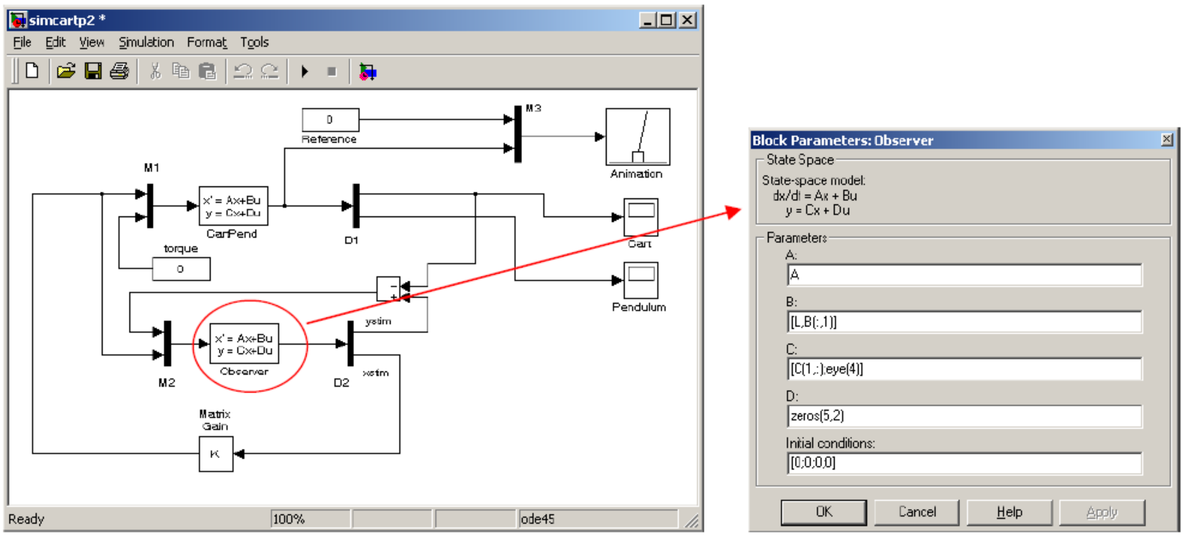
1. Assuming $x_1 = \delta p$, $x_2 = \delta\dot{p}$, $x_3 = \delta\vartheta$, $x_4 = \delta\dot{\vartheta}$, $u_1 = \delta F$, $u_2 = \delta\tau$, $y_1 = \delta p$, $y_2 = \delta\vartheta$, determine matrices $\mathbf{A}$, $\mathbf{B}$, $\mathbf{C}$, $\mathbf{D}$ of the linearised system.

2. Verify that the linearised system is completely controllable and completely observable using as input and output force and cart position, respectively, and is not completely controllable and not completely observable using as input and output torque and beam angular position, respectively.

3. Design a full-state feedback control law, based on the input δF, that places the poles of the closed-loop system at the roots of the polynomial $\chi^o(s) = \left(s^2 + 1.5s + 1\right)\left(s^2 + 2s + 1\right)$.

4. Design an observer, based on the output δp, that places the poles of the error system at the roots of the polynomial $\chi^o(s) = \left(s^2 + 15s + 100\right)\left(s^2 + 20s + 100\right)$.

5. Using the following Simulink diagram, simulate the behaviour of the system starting from a perturbed initial condition (e.g., a beam initial position different from zero).



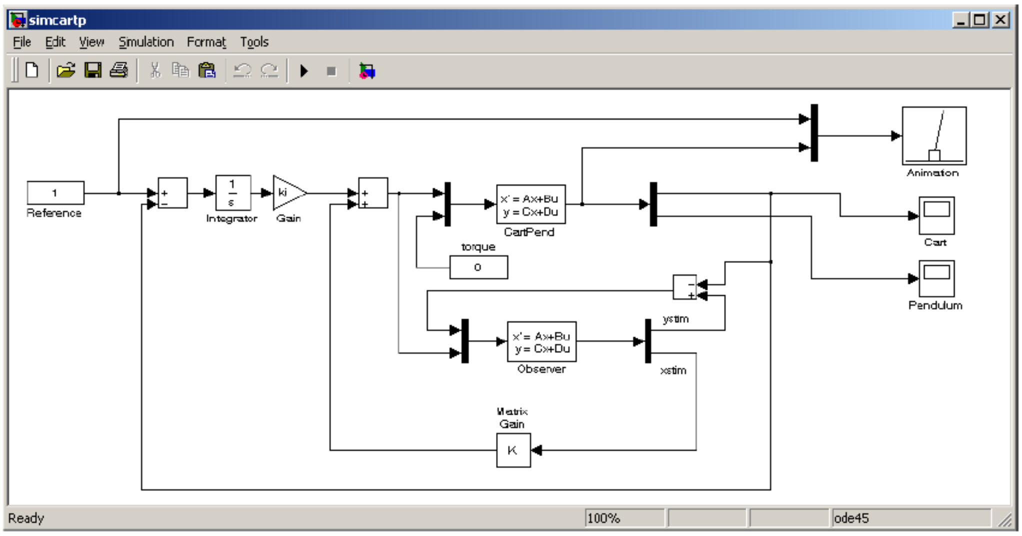
Remarks:

- Animation block can be copied by the `penddemo` Simulink diagram. This block requires a block `reference` constituted by a constant value.

- The parameters required by the mux and demux blocks to specify the number of inputs and outputs are:
  M1:2; M2:2; M3:[1;2]; D1:2; D2:[1;4].
- To set an initial condition different from zero one can use the parameter initial condition of the block CartPend.

6. Design a full-state feedback that allows to track a reference signal, placing the poles of the augmented system at the roots of the polynomial $\chi^o(s) = \left(s^2 + 1.5s + 1\right)\left(s^2 + 2s + 1\right)(s + 2)$.

7. Using the following Simulink diagram, simulate the response of the system to a step on the reference signal.



Control System Toolbox - Useful functions	
`K = ctrb(A,B)`	Compute the controllability matrix associated to (A, B)
`K = obsv(A,C)`	Compute the observability matrix associated to (A, C)
`K = place(A,B,p)`	Compute the gain that places the poles of (A, B) at p