

Automatic Control A

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A.A. 2016/2017 – September 7, 2017



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This file consists of **8** pages (including cover).

During the exam you are not allowed to exit the room for any other reason than handing your work or withdrawing from the exam.

You are not allowed to withdraw from the exam during the first 30 minutes.

During the exam you are not allowed to consult books or any kind of notes.

You are not allowed to use calculators with graphic display.

Solutions and answers can be given **either in English or in Italian**.

Solutions and answers must be given **exclusively in the reserved space**. Only in the case of corrections, or if the space is not sufficient, use the back of the front cover.

The clarity and the order of the answers will be considered in the evaluation.

At the end of the test you have to **hand this file only**. Every other sheet you may hand will not be taken into consideration.



Exercise 1

Consider the following nonlinear dynamical system

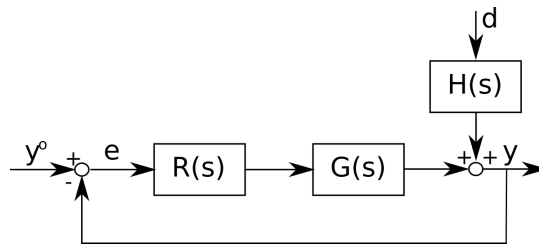
$$\begin{cases} \dot{x}_1 = -x_1 + x_2 e^u \\ \dot{x}_2 = -x_1 + u e^{x_2} \\ \dot{x}_3 = e^{x_1} - x_3 + u \\ y = x_1 e^{x_2} + x_2 e^{x_1} \end{cases}$$

1. Write the expression of the linearised system for the equilibrium $\bar{x} = [0 \ 0 \ 1]^T$, $\bar{u} = 0$, $\bar{y} = 0$.

2. Is this equilibrium stable, unstable, or asymptotically stable?

Exercise 2

Consider the following control loop



where $G(s) = \frac{1 - 0.5s}{(1 + 5s)(1 + 50s)}$ and $H(s) = \frac{5}{1 + s}$.

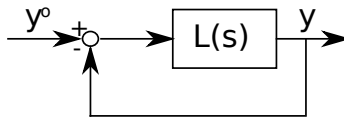
1. Compute the transfer function $R(s)$ of a **PID** controller in such a way that:

- $|e_\infty| = 0$ for $y^\circ(t) = \text{sca}(t)$ and $d(t) = \text{sca}(t)$;
- $\varphi_m \geq 70^\circ$ and ω_c is approximately maximised.

2. Can we use the closed-loop Ziegler and Nichols rule to automatically tune the PID regulator in step 1 if the process is represented by the previous $G(s)$ transfer function?

Exercise 3

Consider the following closed-loop system



where $L(s) = \rho \frac{s - 1}{(s - 2)(s^2 + 2s + 1)}$.

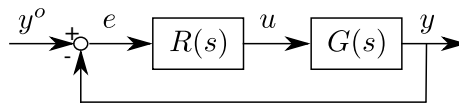
1. Sketch the direct and inverse root loci.

2. Using the previous root loci, find the values of ρ for which the closed-loop system is asymptotically stable.

3. Verify the result in step 2 using the Routh criterion.

Exercise 4

Consider the following control loop



where $R(s) = \frac{1+s}{s}$ and $G(s) = \frac{10}{1+s}$.

1. Determine a sampling time for the digital implementation of $R(s)$, in such a way that the decrement of phase margin introduced by the sample-and-hold is equal to 5 deg.

2. Determine the transfer function $R(z)$ of the digital regulator using the implicit Euler transformation.

3. Write the algorithm that implements the digital controller derived in step 2.

Exercise 5

1. Clearly explain:

- when using a cascaded control architecture can give better performance with respect to a single PID loop;
- which are the rules to separate the process model into two subprocesses, one controlled by the inner and one controlled by the outer regulator.

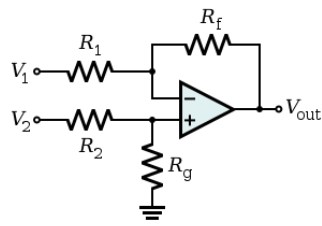
2. For each of the following transfer functions:

$$G_1(s) = \frac{(1+s)(1-s)}{(s+10)(s+11)} \quad G_2(s) = \frac{10}{(1+100s)(1+s)}$$

write the transfer functions that have to be considered to tune the inner ($G_{1_{in}}(s)$, $G_{2_{in}}(s)$) and the outer ($G_{1_{out}}(s)$, $G_{2_{out}}(s)$) regulators.

Exercise 6

Consider the following circuit



1. Compute the input-output relation that describes the circuit.
2. Determine the values of the resistors so that the previous circuit acts as a differential amplifier with gain equal to 10.
3. Explain the differences between single-ended and differential signaling, highlighting pros and cons of each methodology.