Control of Industrial and Mobile Robots

Prof. Rocco, Bascetta

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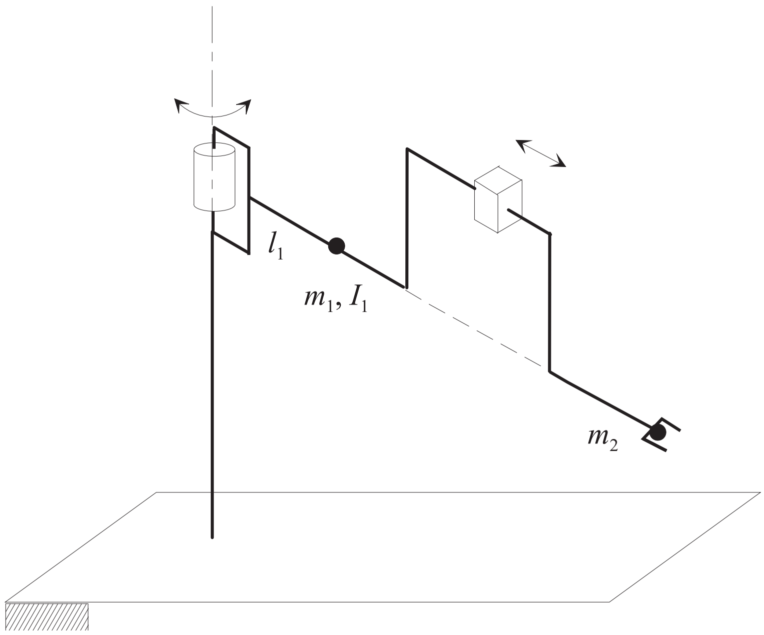
SOLUTION

Control of Industrial and Mobile Robots
Prof. Luca Bascetta and Prof. Paolo Rocco

EXERCISE 1

1. Consider the manipulator sketched in the picture, where the mass of the second link is assumed to be concentrated at the end-effector:



Find the expression of the inertia matrix $\mathbf{B}(\mathbf{q})$ of the manipulator.

NOTHING

2. Compute the matrix $\mathbf{C}(\mathbf{q}, \dot{\mathbf{q}})$ of the Coriolis and centrifugal terms[1] for this manipulator.

NOTHING

3. Write the dynamic model for this manipulator.

NOTHING

[1]The general expression of the Christoffel symbols is $c_{ijk} = \frac{1}{2}\left(\frac{\partial b_{ij}}{\partial q_k} + \frac{\partial b_{ik}}{\partial q_j} - \frac{\partial b_{jk}}{\partial q_i}\right)$

4. For a generic manipulator without gravitational and friction effects, show that the equation:

$$\dot{\mathbf{q}}^T \left(\dot{\mathbf{B}}(\mathbf{q}) - 2\mathbf{C}(\mathbf{q}, \dot{\mathbf{q}})\right) \dot{\mathbf{q}} = 0$$

is valid for any choice of the Coriolis and centrifugal matrix $\mathbf{C}(\mathbf{q}, \dot{\mathbf{q}})$.

NOTHING

EXERCISE 2

Consider a kinematically redundant manipulator.

1. Write the general expression of the solutions of the inverse kinematics problem at velocity level.

NOTHING

2. Consider the weighted pseudo inverse method for the solution of the inverse kinematics of a redundant manipulator. Write the expression of the cost function and discuss a criterion to select the weights.

NOTHING

3. The inverse kinematics at velocity level for a redundant manipulator is often implemented in closed-loop. Explain the reason for this and sketch the block diagram for such a closed-loop scheme.

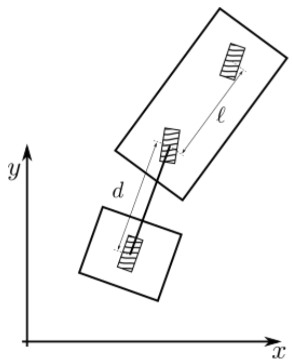
NOTHING

4. Consider now motion planning of the end-effector position. If the end-effector task is expressed in terms of position only, what is the minimum number of joints for the manipulator to be redundant with respect to this task?

NOTHING

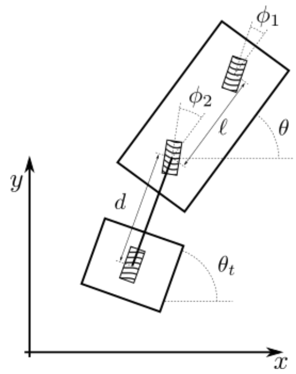
EXERCISE 3

Consider a car-trailer system, constituted by a trailer equipped with a fixed wheel and a car equipped with two steering wheels, shown in the figure below.



1. Determine the configuration vector and show the configuration variables on the figure above.

   The car-trailer robot configuration, with reference to the picture below, is represented by vector $\mathbf{q} = [x, y, \theta, \theta_t, \phi_1, \phi_2]$, where (x, y) is the position of the car rear wheel contact point.



2. Derive the kinematic constraints that allow to determine the kinematic model of the car-trailer system, and write them in Pfaffian form.

   We can write the pure rolling constraints referred to each wheel of the car and of the trailer as follows

$$\dot{x}_1 \sin(\theta + \phi_1) - \dot{y}_1 \cos(\theta + \phi_1) = 0 \tag{1}$$
$$\dot{x} \sin(\theta + \phi_2) - \dot{y} \cos(\theta + \phi_2) = 0 \tag{2}$$
$$\dot{x}_t \sin(\theta_t) - \dot{y}_t \cos(\theta_t) = 0 \tag{3}$$

   where (x_1, y_1) is the position of the car front wheel contact point, and (x_t, y_t) the position of the trailer wheel contact point.
   We can relate the position of the front wheel contact point and of the trailer wheel contact point to (x, y) through a rigidity constraint

$$x_1 = x + \ell \cos\theta$$
$$y_1 = y + \ell \sin\theta$$

   and

$$x_t = x - d \cos\theta_t$$
$$y_t = y - d \sin\theta_t$$

   Differentiating the two relations with respect to time we obtain

$$\dot{x}_1 = \dot{x} - \ell\dot{\theta} \sin\theta$$
$$\dot{y}_1 = \dot{y} + \ell\dot{\theta} \cos\theta$$

   and

$$\dot{x}_t = \dot{x} + d\dot{\theta}_t \sin\theta_t$$
$$\dot{y}_t = \dot{y} - d\dot{\theta}_t \cos\theta_t$$

Substituting these relations in (1) and (3) we obtain

$$\begin{aligned} \dot{x}\sin(\theta+\phi_1) - \dot{y}\cos(\theta+\phi_1) - \ell\dot{\theta}\cos\phi_1 &= 0 \\ \dot{x}\sin(\theta+\phi_2) - \dot{y}\cos(\theta+\phi_2) &= 0 \\ \dot{x}\sin(\theta_t) - \dot{y}\cos(\theta_t) - d\dot{\theta}_t &= 0 \end{aligned}$$

The three constraints that describe the car-trailer can be written in Pfaffian form as

$$A^T(\mathbf{q})\,\dot{\mathbf{q}} = \begin{bmatrix} \sin(\theta+\phi_1) & -\cos(\theta+\phi_1) & -\ell\cos\phi_1 & 0 & 0 & 0 \\ \sin(\theta+\phi_2) & -\cos(\theta+\phi_2) & 0 & 0 & 0 & 0 \\ \sin(\theta_t) & -\cos(\theta_t) & 0 & -d & 0 & 0 \end{bmatrix} \begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \\ \dot{\theta}_t \\ \dot{\phi}_1 \\ \dot{\phi}_2 \end{bmatrix} = 0$$

3. Consider the following equations

$$\begin{aligned} \dot{x} &= v\,d\ell\cos(\phi_1)\cos(\theta_t+\phi_2) \\ \dot{y} &= v\,d\ell\cos(\phi_1)\sin(\theta_t+\phi_2) \\ \dot{\theta} &= v\,d\sin(\phi_1-\phi_2) \\ \dot{\theta}_t &= v\,\ell\cos(\phi_1)\sin(\theta_t-\theta-\phi_2) \\ \dot{\phi}_1 &= \omega_1 \\ \dot{\phi}_2 &= \omega_2 \end{aligned}$$

Is this the kinematic model of the car-trailer system? Clearly motivate the answer and support it with a theoretical proof.

First of all, matrix

$$\begin{bmatrix} d\ell\cos(\phi_1)\cos(\theta_t+\phi_2) & 0 & 0 \\ d\ell\cos(\phi_1)\sin(\theta_t+\phi_2) & 0 & 0 \\ d\sin(\phi_1-\phi_2) & 0 & 0 \\ \ell\cos(\phi_1)\sin(\theta_t-\theta-\phi_2) & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

has rank equal to 3, and thus the three vectors are linearly independent.
It is then easy to check that the three columns of the previous matrix are vectors in the null of $A^T(\mathbf{q})$.

4. Consider now the car without the trailer, and assume the rear wheel is fixed while the front is steerable. The car velocity and front steering position are constrained as follows

$$0 \le v \le \bar{v} \qquad \bar{\phi}_m \le \phi \le \bar{\phi}_M$$

How should we limit the linear and angular velocity of the canonical model, in order to be consistent with these constraints?

The relation between the steering angle and the unicycle velocities is

$$\tan \phi = \frac{\omega v}{L}$$

Then constraint

$$\bar{\phi}_m \leq \phi \leq \bar{\phi}_M$$

can be transformed into

$$\tan \bar{\phi}_m \leq \tan \phi \leq \tan \bar{\phi}_M$$

and

$$v \frac{\tan \bar{\phi}_m}{L} \leq \omega \leq v \frac{\tan \bar{\phi}_M}{L}$$

For the linear velocity the constraint remains unchanged

$$0 \leq v \leq \bar{v}$$

EXERCISE 4

1. Two important parts of an autonomous navigation system are the local and the global planner. What are the most important characteristics of these two functionalities?

   None

2. Explain what are the advantages of structuring the navigation system in a hierarchical way, separating local from global planning.

   None

3. Consider now as global planner RRT*. Write the pseudocode of the rewire procedure, and explain the role of this procedure inside the RRT* algorithm and its connection with optimality.

   None

4. RRT* is used to plan the path of an electric vehicle, whose battery pack can be recharged when the vehicle travels a hill. Can the function

   $$c(\sigma) = \text{length}(\sigma) + \text{discharge}(\sigma)$$

   where σ is a path, lenth is the function that computes the length of a path, and discharge the function that computes the battery discharge (or charge if the value is negative) that occurs travelling a path, be used as cost for the planner?

None