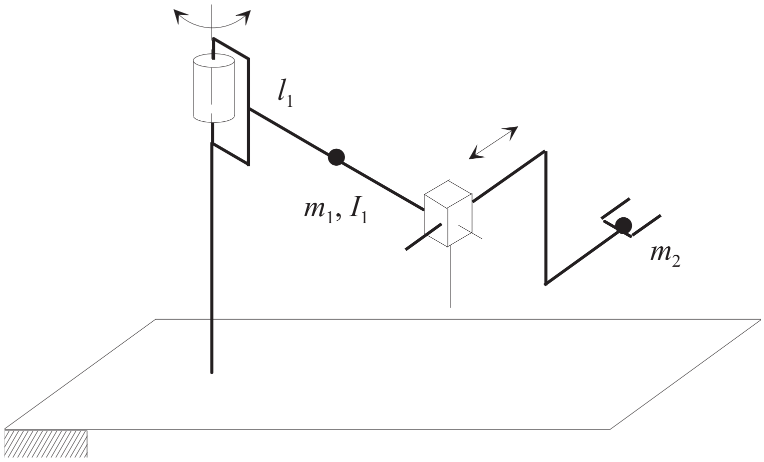
EXERCISE 1

1. Consider the manipulator sketched in the picture, where the mass of the second link is assumed to be concentrated at the end-effector:



Find the expression of the inertia matrix $\mathbf{B}(\mathbf{q})$ of the manipulator.

2. Compute the matrix $\mathbf{C}(\mathbf{q}, \dot{\mathbf{q}})$ of the Coriolis and centrifugal terms[1] for this manipulator.

3. Check that matrix $\mathbf{N}(\mathbf{q}, \dot{\mathbf{q}}) = \dot{\mathbf{B}}(\mathbf{q}) - 2\mathbf{C}(\mathbf{q}, \dot{\mathbf{q}})$ is skew symmetric.

4. Cite one case in robotics where the property that matrix $\mathbf{N}(\mathbf{q}, \dot{\mathbf{q}})$ is skew symmetric is used.

[1] The general expression of the Christoffel symbols is $c_{ijk} = \frac{1}{2}\left(\frac{\partial b_{ij}}{\partial q_k} + \frac{\partial b_{ik}}{\partial q_j} - \frac{\partial b_{jk}}{\partial q_i}\right)$

EXERCISE 2

1. Consider the generation of a position trajectory in the Cartesian space. Select as an initial point $\mathbf{p}_i = \begin{bmatrix} 3 \\ 1 \\ 1 \end{bmatrix}$ and as a final point $\mathbf{p}_f = \begin{bmatrix} 0 \\ 1 \\ 5 \end{bmatrix}$. Write the expression of a segment connecting the initial and the final points, parameterized with the natural coordinate.

2. Assume a travel time $T = 2s$. Design a trajectory, which covers the path determined in the previous step, using a cubic dependence on time.

3. Compute the maximum linear velocity of the end effector along the trajectory designed in the previous step. Check whether this maximum value exceeds a maximum admissible velocity of 2 m/s. In case, explain (without going through the computations) how you would modify the trajectory generation.

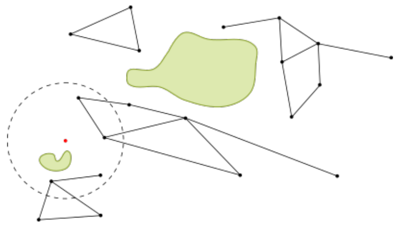
4. Suppose that the manipulator is kinematically redundant for the task of end effector positioning. Write the expression of the inverse kinematics based on the *weighted* pseudo-inverse matrix and explain what is the optimization problem solved with this approach. Also explain what might be the reason to use the weighted pseudo-inverse instead of the standard pseudo-inverse matrix.

EXERCISE 3

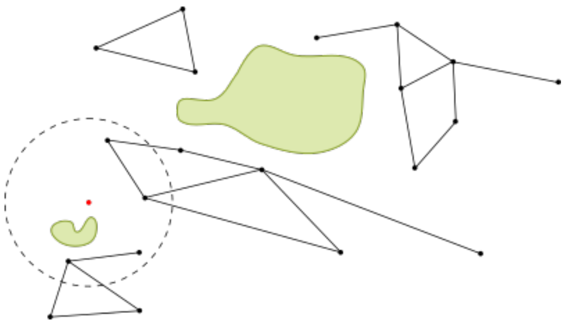
1. Write and explain the pseudocode of the algorithm to construct the probabilistic roadmap used by PRM.

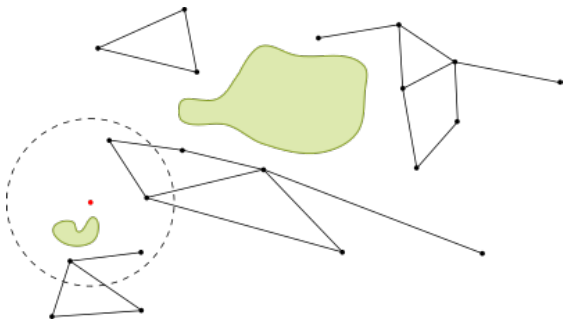
2. Explain how the previous algorithm has to be modified in order to obtain sPRM and PRM* algorithms.

3. After the execution of some iterations of PRM algorithm we have the situation depicted below



where the black dots are the nodes sampled in previous iterations, the red dot is the node sampled in the actual iteration, the green blobs are obstacles and the black dashed circle is the neighbourhood considered to defined the Near set.
Draw in the left picture the result of an iteration of PRM and in the right picture the result of an iteration of sPRM.





4. What is the main difference between PRM (or sPRM or PRM*) and RRT (or RRT*)?

EXERCISE 4

1. What is the *canonical simplified model for nonholonomic mobile robots*? Why is it important in the context of designing a controller for a nonholonomic robot?

2. Show how the unicycle, differential drive, and rear-wheel drive bicycle kinematic models can be made equivalent to the canonical model.

3. Consider a bicycle kinematic model without reverse. Show how the actuation constraints $0 \leq v \leq v_M$ and $-\phi_M \leq \phi \leq \phi_M$ can be rewritten in terms of the canonical model input variables.

4. Considering a reference point P on the chassis of the robot, derive a control law for the canonical model that allows to reduce it to two independent double integrators.