Automatic Control A

(Prof. Bascetta)

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Name:...

University ID number:...............................

Signature:..

Warnings:

- This file consists of **8** pages (including cover). All the pages should be signed.
- During the exam you are not allowed to exit the room for any other reason than handing your work or withdrawing from the exam.
- You are not allowed to withdraw from the exam during the first 30 minutes.
- During the exam you are not allowed to consult books or any kind of notes.
- You are not allowed to use calculators with graphic display.
- Solutions and answers can be given **either in English or in Italian**.
- Solutions and answers must be given **exclusively in the reserved space**. Only in the case of corrections, or if the space is not sufficient, use the back of the front cover.
- The clarity and the order of the answers will be considered in the evaluation.
- At the end of the test you have to **hand this file only**. Every other sheet you may hand will not be taken into consideration.

Use this page ONLY in case of corrections or if the space reserved for some answers turned out to be insufficient

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Exercise 1

Consider the following dynamical system

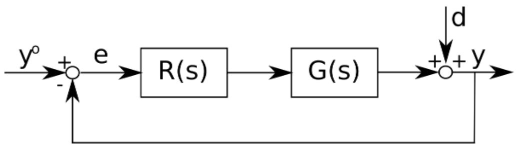
$$\begin{cases} \dot{x}_1 = -x_1 + x_2 + \alpha u \\ \dot{x}_2 = -x_2 + 2\alpha u \\ y = (1-\alpha)x_1 + x_2 \end{cases}$$

1.1 Find the values of parameter $\alpha \in \mathbb{R}$ for which the system is completely controllable and completely observable.

1.2 Assuming $\alpha = 1$, draw the block diagram showing the decomposition of the system into its controllable/not controllable and observable/not observable parts (Kalman decomposition).

Exercise 2

Consider the following control system



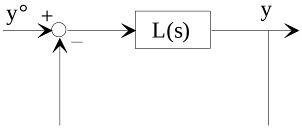
where $G(s) = 0.1\frac{1-0.1s}{1+0.01s}$.

Compute the transfer function $R(s)$ of a regulator in such a way that:

- $|e_{\infty}| = 0$ for $y°(t) = 5\,\mathrm{sca}(t)$ and $d(t) = 0$;
- a disturbance $d(t) = A\sin(\omega t)$, where A is an arbitrary constant and $\omega \leq 0.01\ rad/s$, is attenuated on the output y 1000 times;
- the phase margin φ_m is greater or equal to 75°;
- the crossover frequency ω_c is greater or equal to 0.1 *rad/s*.

Exercise 3

Consider the following closed-loop system



where $L(s) = \rho \dfrac{0.5(s-3)}{(1+s)(1+0.5s)(s+3)}$.

3.1 Sketch the direct and inverse root loci.

3.2 Using the previous root loci, find the values of ρ for which the closed-loop system is asymptotically stable.

Exercise 4

Consider the general discrete time linear time invariant dynamical system

$$\begin{cases} x(k+1) = Ax(k) + Bu(k) \\ y(k) = Cx(k) \end{cases}$$

4.1 Write the expressions of the state and output zero-input response and zero-state response generated by an initial state x_0 and an input $u(k), k \geq 0$.

4.2 Given the discrete time system

$$\begin{cases} x_1(k+1) = 2x_2(k) \\ x_2(k+1) = -2x_3(k) \\ x_3(k+1) = u(k) \\ y(k) = x_1(k) - x_2(k) + 2x_3(k) \end{cases}$$

show that the zero-input response for $k \geq 3$ is always equal to zero, whatever the value of the initial state is.

4.3 Compute the transfer function. Is the discrete time system stable, unstable or asymptotically stable?

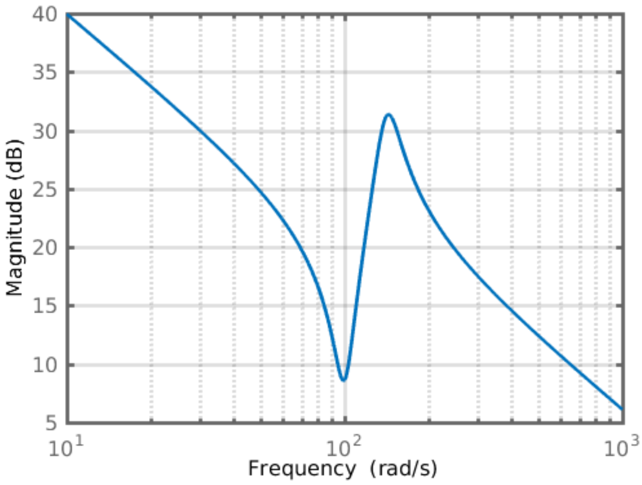
Exercise 5

Consider a servomechanism with elastic transmission characterized by the following parameters:

- reduction ratio $n = 100$
- motor moment of inertia $J_m = 5 \cdot 10^{-4}\ Kg\ m^2$

5.1 A set of experiments performed on the servomechanism allows to compute the Bode diagram of the frequency response of the transfer function from motor torque to motor velocity.

Using the Bode diagram compute an estimate of the load moment of inertia J_l and of the transmission elastic constant K_{el}.



5.2 Design a PI velocity controller, accurately describing the procedure, that maximizes the damping of the oscillations generated by the elastic transmission.

Exercise 6

6.1 Draw the circuit diagram of a sample-and-hold circuit.

6.2 Describe how the circuit works. When and why is the sample-and-hold circuit required?

6.3 Assume that an analog sinusoidal signal, with a frequency of 500 Hz, has to be converted into a digital signal using an ADC with 10 bit and without sample-and-hold circuit. Which is the maximum value of the conversion interval?