Control of Industrial and Mobile Robots

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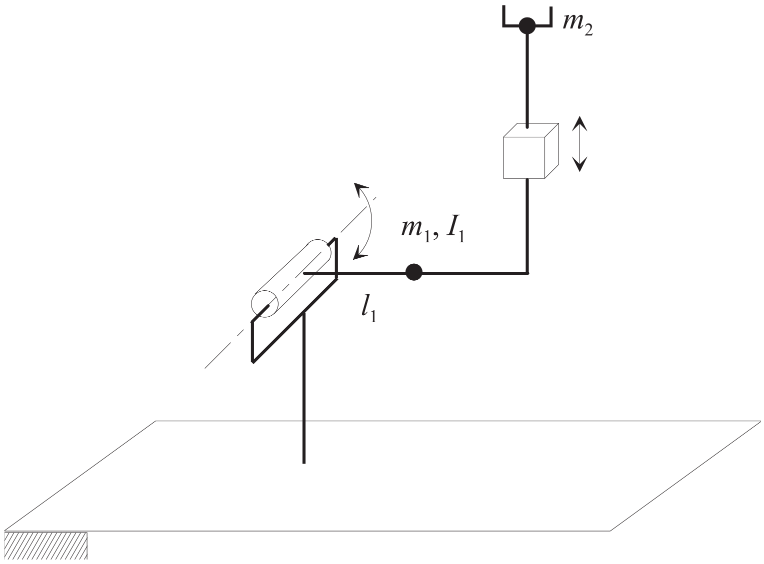
Warnings

- This file consists of **10** pages (including cover).
- During the exam you are not allowed to exit the room for any other reason than handing your work or withdrawing from the exam.
- You are not allowed to withdraw from the exam during the first 30 minutes.
- During the exam you are not allowed to consult books or any kind of notes.
- You are not allowed to use calculators with graphic display.
- Solutions and answers can be given **either in English or in Italian**.
- Solutions and answers must be given **exclusively in the reserved space**. Only in the case of corrections, or if the space is not sufficient, use the back of the front cover.
- The clarity and the order of the answers will be considered in the evaluation.
- At the end of the test you have to **hand this file only**. Every other sheet you may hand will not be taken into consideration.

EXERCISE 1

1. Consider the manipulator sketched in the picture, where the mass of the second link is assumed to be concentrated at the end-effector:



Find the expression of the inertia matrix $\mathbf{B}(\mathbf{q})$ of the manipulator.

2. Compute the matrix $\mathbf{C}(\mathbf{q}, \dot{\mathbf{q}})$ of the Coriolis and centrifugal terms[1] for this manipulator.

3. Check that the expression $\dot{\mathbf{q}}^T \mathbf{N}(\mathbf{q}, \dot{\mathbf{q}})\dot{\mathbf{q}} = 0$, where $\mathbf{N}(\mathbf{q}, \dot{\mathbf{q}}) = \dot{\mathbf{B}}(\mathbf{q}) - 2\mathbf{C}(\mathbf{q}, \dot{\mathbf{q}})$, is true in this case.

4. Disregarding the gravitational terms, write the dynamic model of this manipulator in a form that is linear with respect to a set of dynamic parameters.

[1]The general expression of the Christoffel symbols is $c_{ijk} = \frac{1}{2}\left(\frac{\partial b_{ij}}{\partial q_k} + \frac{\partial b_{ik}}{\partial q_j} - \frac{\partial b_{jk}}{\partial q_i}\right)$

EXERCISE 2

1. Suppose that a trajectory for a scalar variable has to be defined, which achieves the values reported in the following table, at the given instants:

$$\begin{array}{ccccc} t_1 = 0 & t_2 = 2 & t_3 = 5 & t_4 = 8 & t_5 = 10 \\ q_1 = 20 & q_2 = 0 & q_3 = 20 & q_4 = 40 & q_5 = 50 \end{array}$$

   Assume that such points are interpolated by a single polynomial of suitable degree. Using the specific data of this exercise, write the equation in matrix form that allows to compute the coefficients of such polynomial.

2. Assume now that you want to use cubic polynomials in each interval. Assign suitable values to the speed at the intermediate points.

3. Using the values of speed previously evaluated, compute the expression of the cubic polynomial for the first interval (from t_1 to t_2).

4. What are the maximum values (in terms of absolute values) of the speed and of the acceleration for the cubic profile determined at the previous step?

EXERCISE 3

Consider a rear-wheel drive bicycle robot under the following assumptions:

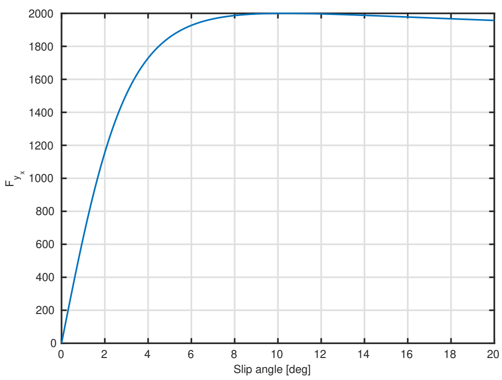
- braking force can be neglected;
- sideslip and steering angles are small (i.e., $\cos x \approx 1$ and $\sin x \approx x$);
- force-slip relation is linear;
- the absolute velocity is slowly varying.

1. Write the configuration vector and the equations describing the kinematic model. Explain the meaning of each symbol used in the equations.

2. Write the equations describing the dynamic model considering the previous assumptions and using as state variables sideslip and yaw rate. Explain the meaning of each symbol used in the equations.

3. Consider now a force-slip relation for the lateral force that is linear up to the maximum available force (given by friction), and then saturates to the friction force. Write the expression of the front/rear lateral forces and the equations describing the dynamic model of the vehicle, based on this force-slip relation. Explain the meaning of each symbol used in the equations.

4. Determine the parameters of the force-slip relation introduced in the previous step according to the figure below, assuming $F_z = 2000\,\mathrm{N}$ and $\mu = 1$.



EXERCISE 4

1. What is the *canonical simplified model for nonholonomic mobile robots*? Why is it important in the context of designing a controller for a nonholonomic robot?

2. Show how the unicycle, differential drive, and rear-wheel drive bicycle kinematic models can be made equivalent to the canonical model.

3. Consider a bicycle kinematic model without reverse. Show how the actuation constraints $0 \leq v \leq v_M$ and $-\phi_M \leq \phi \leq \phi_M$ can be rewritten in terms of the canonical model input variables.

4. Considering a reference point P on the chassis of the robot, derive a control law for the canonical model that allows to reduce it to two independent double integrators.