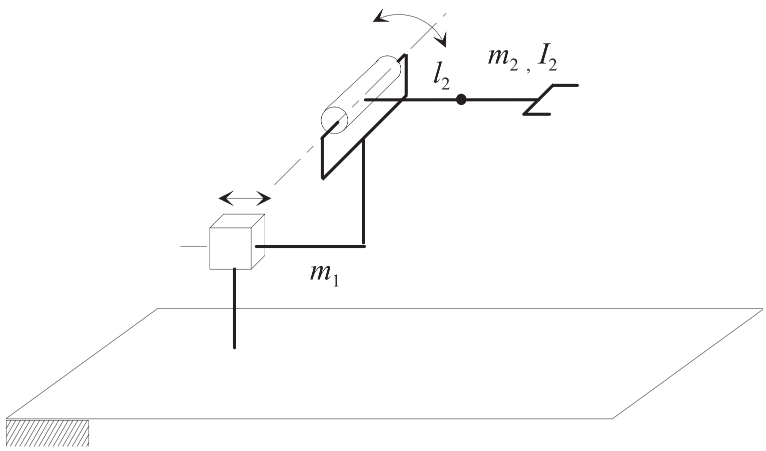
EXERCISE 1

1. Consider the manipulator sketched in the picture:



Find the expression of the inertia matrix $\mathbf{B}(\mathbf{q})$ of the manipulator[1]

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[1]The cross product between vector $a = \begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix}$ and $b = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$ is $c = a \times b = \begin{bmatrix} a_2 b_3 - a_3 b_2 \\ a_3 b_1 - a_1 b_3 \\ a_1 b_2 - a_2 b_1 \end{bmatrix}$

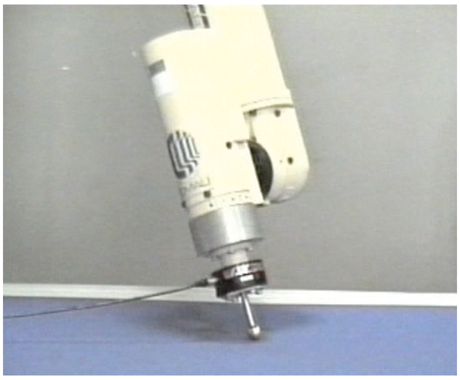
2. Ignoring the Coriolis and centrifugal terms, write the dynamic model of the manipulator.

3. Show that the dynamic model is linear with respect to a certain set of dynamic parameters.

4. Write the expression of a "PD + gravity compensation" control law in the joint space for this specific manipulator.

EXERCISE 2

1. Consider an interaction task of a manipulator, with a frictionless and rigid surface, as in this picture:



   Assume a point contact and draw a contact frame directly on the picture. Based on this frame and neglecting angular velocities and moments, express the natural and the artificial constraints for this problem, and specify the selection matrix.

2. Explain what an implicit force controller is and why it might be convenient with respect to an explicit solution.

3. Suppose now that along the force controlled direction an explicit force controller has to be designed. Sketch the block diagram of such controller and design it taking a bandwidth of 30 rad/s.

4. Repeat the process in case an implicit force controller, for the same bandwidth, has to be designed.

EXERCISE 3

1. Consider a wheel rolling without slipping on the horizontal plane, keeping the sagittal plane in the vertical direction. Write the expression of the pure rolling constraint in the case of a steerable wheel, explaining its physical meaning.

2. Show that the previous kinematic constraint is a nonholonomic constraint.

3. Consider now the dynamics of a steerable rolling wheel, describe the two most important modelling approaches that can be used to represent the wheel-ground interaction (longitudinal and lateral) forces stressing their differences and/or similarities.

4. Describe a linear model to represent the wheel lateral force.

EXERCISE 4

1. Consider a robot represented by a bicycle kinematic model. Describe an algorithm that allows to find a trajectory (only the expressions of $x(t)$ and $y(t)$ are required), feasible with respect to the kinematic model, to move the robot in an obstacle free environment from an initial state $q_i = [x_i \; y_i \; \theta_i \; v_i]$ at $t_i = 0$ to a final state $q_f = [x_f \; y_f \; \theta_f \; v_f]$ at $t_f = \bar{t}_f$ (where the value of $\bar{t}_f$ is known), exploiting the flatness property.

2. How can the same planning problem be solved, considering obstacles as well, using a sampling-based planning algorithm?

3. Consider the following bicycle kinematic model

$$\dot{x} = v\cos\theta \qquad \dot{y} = v\sin\theta \qquad \dot{\theta} = \frac{v}{\ell}\tan\phi$$

and a point P related to the bicycle rear wheel contact point (x, y) by the following relations

$$x_P = x + \varepsilon\cos\theta \qquad y_P = y + \varepsilon\sin\theta$$

Show how a feedback control law that linearises the bicycle model can be derived.

4. Explain why the control system designed in the previous step cannot be used to regulate the pose of the robot.