Automatic Control - Laboratory 3
Discrete time and digital control systems
Prof. Luca Bascetta

Exercise 1

Consider the following LTI discrete time system

$$\begin{aligned} x_1(k+1) &= 0.5x_1(k) + 0.5x_2(k) + 0.5u(k) \\ x_2(K+1) &= -0.5x_1(k) + 0.5x_2(k) + 0.5u(k) \\ y(k) &= x_1(k) + x_2(k) + u(k) \end{aligned}$$

1. Plot poles and zeros in the complex plane, and verify the asymptotic stability of the system.
2. Compute the static gain.
3. Plot the unity step response, and verify that it asymptotically tends to the value of the static gain.

Exercise 2

Consider the following transfer function

$$G(z) = \frac{1-p}{z-p}$$

1. Plot the unity step response for $p = 0.3$, $p = 0.6$, $p = 0.9$, $p = -0.3$.
2. Select one of the previous values for p, and determine the frequency response of the system.
3. With the same value, verify that the response of the system to $u(k) = \sin(k)$ is coherent with the characteristics of the sinusoidal response determined using the sinusoidal response theorem.

Exercise 3

Consider the Shannon formula

$$v(t) = \sum_{k=-\infty}^{+\infty} \left[v^*(k) \frac{\sin(\Omega_N t - k\pi)}{\Omega_N t - k\pi} \right]$$

Assume that the following signal

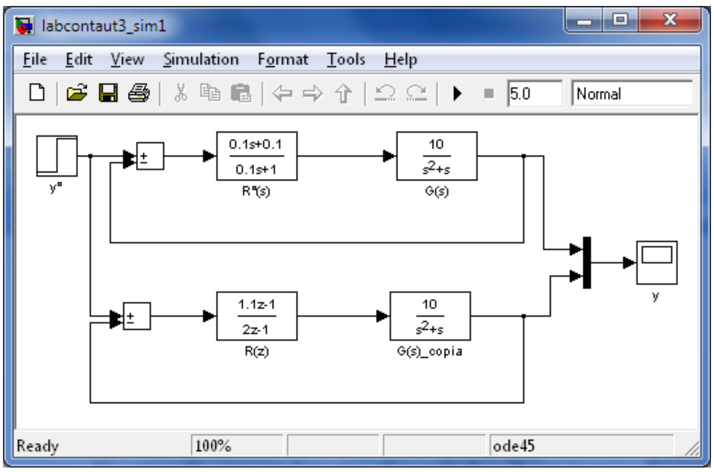
$$v(t) = \sin(t) + \sin(2t)$$

were sampled with a sampling time $T = 1$ and that samples for $k \in [-50, 50]$ are available.
Using Shannon formula compute $v(0.3)$ and compare the result with its exact value.
Plot the contribution of each addendum

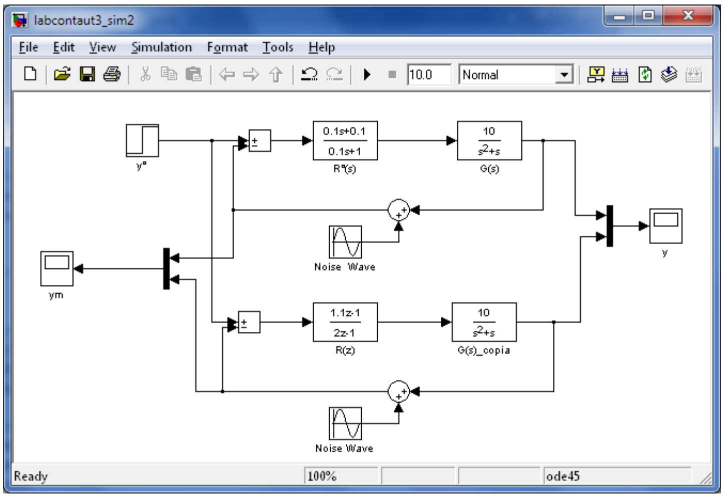
$$v^*(k) \frac{\sin(\Omega_N t - k\pi)}{\Omega_N t - k\pi} \qquad k \in [-50, 50]$$

Exercise 4

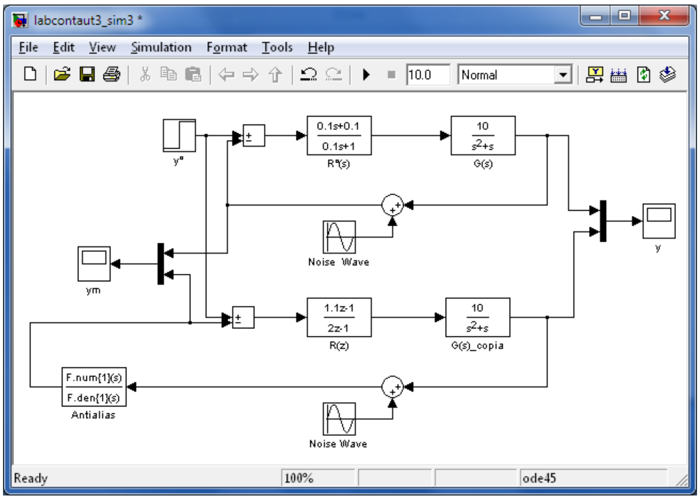
Using Simulink, simulate the digital control system shown in the picture below (sampling time $T = 0.1\,s$), and compare it with the corresponding analogue control system.



Analyse the effect of a sinusoidal disturbance in the feedback path, amplitude 0.05 and frequency 500 rad/s, on the digital and analogue control systems.



Design an anti-aliasing filter for the digital control system and simulate the closed-loop system together with the filter.



Control System Toolbox - Useful functions

`sist = ss(A,B,C,D,-1)`	Define a state-space system given A, B, C, D matrices
`sist = tf(num,den,-1)`	Define a transfer function given numerator and denominator coefficients
`sist = zpk(z,p,k,-1)`	Define a transfer function given zero/pole vectors and gain
`sistd=c2d(sistc,Tc,'zoh')`	Transform a continuous time system to a discrete time one using ZOH transformation
`sistd=c2d(sistc,Tc,'tustin')`	Transform a continuous time system to a discrete time one using Tustin transformation