Control of Mobile Robots

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September 9, 2021

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Warnings

- This file consists of **8** pages (including cover).
- During the exam you are not allowed to exit the room for any other reason than handing your work or withdrawing from the exam.
- You are not allowed to withdraw from the exam during the first 30 minutes.
- During the exam you are not allowed to consult books or any kind of notes.
- You are not allowed to use calculators with graphic display.
- Solutions and answers can be given **either in English or in Italian**.
- Solutions and answers must be given **exclusively in the reserved space**. Only in the case of corrections, or if the space is not sufficient, use the back of the front cover.
- The clarity and the order of the answers will be considered in the evaluation.
- At the end of the test you have to **hand this file only**. Every other sheet you may hand will not be taken into consideration.

EXERCISE 1

1. Given a kinematic constraint in Pfaffian form

$$X(\mathbf{q})\,\dot{x} + Y(\mathbf{q})\,\dot{y} + Z(\mathbf{q})\,\dot{z} = 0$$

where $\mathbf{q} = \begin{bmatrix} x & y & z \end{bmatrix}$ is the configuration vector. Illustrate the necessary and sufficient condition for this constraint to be holonomic.

2. Consider the following kinematic constraints

$$\dot{q}_1 + q_1\dot{q}_2 + \dot{q}_3 = 0 \qquad \dot{q}_1 + \dot{q}_2 + q_1\dot{q}_3 = 0$$

where $\mathbf{q} \in \mathbb{R}^3$ is the configuration vector. Verify, using the necessary and sufficient condition, if each of these constraints by itself is holonomic or nonholonomic.

3. Consider the system of two constraints

$$\dot{q}_1 + q_1\dot{q}_2 + \dot{q}_3 = 0 \qquad \dot{q}_1 + \dot{q}_2 + q_1\dot{q}_3 = 0$$

Does the holonomicity/nonholonomicity of each constraint by itself imply the holonomicity/non-holonomicity of the system of constraints? Is the system of two constraints integrable?

EXERCISE 2

1. Consider a unicycle robot carrying a box full of sand, centred with respect to the robot center of gravity. The robot mass and yaw moment of inertia are equal to 30 Kg and 0.8 Kgm^2, when the box is empty, and 60 Kg and 1.6 Kgm^2, when the box is full. During the motion the box is leaking sand, and the mass and yaw moment of inertia change with time according to the following relations

$$M(t) = 60 - 0.5t \qquad I(t) = \frac{8}{5} - \frac{t}{75}$$

Write the dynamic model of the robot, assuming that there is no relative motion between the sand and the box and that the change of mass is slow compared to the motion of the robot.

2. The robot tire lateral force-slip relations are modelled using a piecewise constant model

$$F_y = \begin{cases} C_\alpha \alpha & \alpha \leq \alpha_{lin} \\ F_{y_{max}} & \alpha > \alpha_{lin} \end{cases}$$

where C_α is the cornering stiffness, α the slip angle, and $\alpha_{lin} = 0.1$ rad. The friction coefficient is $\mu = 0.5$.
Write the lateral force-slip relation at $t_1 = 0$ and $t_2 = 30$.

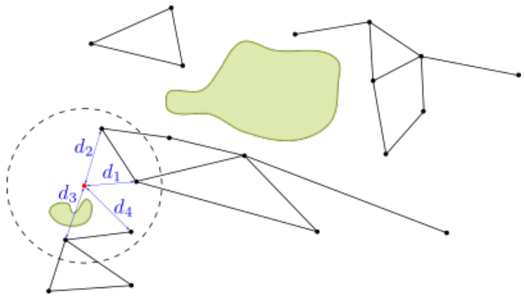
3. Consider that the robot is equipped with wheel velocity controllers tuned for the case of empty box. For the design of a model-based trajectory tracking controller, assuming either the linear and angular velocity or the wheel torque are available to the controller, is it better to consider the kinematic or dynamic model of the robot? Clearly motivate the answer.

EXERCISE 3

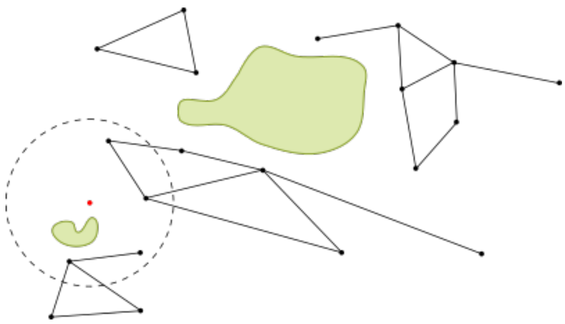
1. Write and explain the pseudocode of the algorithm to construct the probabilistic roadmap used by PRM.

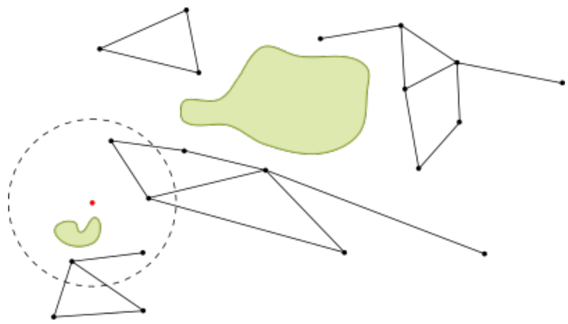
2. Explain how the previous algorithm has to be modified in order to obtain sPRM and PRM* algorithms.

3. After the execution of some iterations of PRM algorithm we have the situation depicted below



where the black dots are the nodes sampled in previous iterations, the red dot is the node sampled in the actual iteration, the green blobs are obstacles, the black dashed circle is the neighbourhood considered to defined the Near set, and d_1, d_2, d_3, d_4 are the distances between the sampled node and each of the nodes in the neighbourhood, with $d_1 < d_2 < d_3 < d_4$.
Draw in the left picture the result of an iteration of PRM and in the right picture the result of an iteration of sPRM.





ESERCIZIO 4

1. Consider the kinematic model of a unicycle robot, and a point P at a distance p from the wheel contact point along the direction of the velocity vector. Write the expression of the feedback linearizing controller and draw the block diagram of the system composed by the robot model and the controller.

2. Show that, applying the feedback linearizing controller the kinematic model of the unicycle is reduced to two independent integrators, i.e.,

$$\dot{x}_P = v_{xP} \qquad \dot{y}_P = v_{yP}$$

3. An experiment is executed on the real robot, performing a step response first on v_{xP} (with $v_{yP} = 0$), and then on v_{yP} (with $v_{xP} = 0$). Due to unmodelled dynamics, the two step responses appears as the response of a first order system (instead of an integrator), with unitary gain and a settling time of 0.05 seconds.
   Design and tune a trajectory tracking controller. Motivate how you select the crossover frequency.